

Expectations, Stagnation and Fiscal Policy: a Nonlinear Analysis*

George W. Evans, University of Oregon and University of St Andrews
Seppo Honkapohja, Aalto University School of Business
Kaushik Mitra, University of Birmingham

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Abstract

Stagnation and fiscal policy are examined in a nonlinear stochastic New-Keynesian model with adaptive learning. There are three steady states. The steady state targeted by policy is locally but not globally stable under learning. A severe pessimistic expectations shock can trap the economy in a stagnation regime, underpinned by a low-level steady state, with falling inflation and output. A large fiscal stimulus may be needed to avoid or emerge from stagnation, and the impacts of forward guidance, credit frictions, central bank credibility and policy delay are studied. Our model encompasses a wide range of outcomes arising from pessimistic expectations shocks.

JEL classification: E62, E63, E52, D84, E71.

Key words: Stagnation Trap, Expectations, Fiscal Policy, Adaptive Learning, New-Keynesian Model.

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1 Introduction

The sluggish macroeconomic performance of advanced market economies in the years following the Great Recession has raised interest in the possibility of the economy becoming stuck for long periods in a distinct stagnation regime associated with the zero lower bound (ZLB) for the policy interest rate.¹ The ongoing global COVID 19 pandemic is currently adding to these longer-term concerns about stagnation. One possible explanation for a stagnation regime is that it is caused by a wide-spread lack of confidence on the part of economic agents. Specifically, a stagnation state with low output, deflation and interest rates constrained by the ZLB can be a self-fulfilling equilibrium of the economy. We develop an extension of a standard new Keynesian (NK) model to account for existence of a stagnation regime – a region of pessimistic expectations that includes a stagnation steady-state. Our analysis assumes that economic agents make forecasts using adaptive learning (AL) and show that in this region expectations become trapped with the stagnation steady state acting as an attractor. Existence of this stagnation regime is consistent with the observation that, under the ZLB constraint, real economic performance of the US, Japanese and the euro area economies appears to be clearly worse than in the earlier period before the ZLB became binding.

Within the context of the standard NK model and rational expectations (RE), the implications of the ZLB have been approached from several angles. First, there is the possibility of exogenous shocks to demand that push the economy to the ZLB. Exogenous discount rate or, more plausibly, credit-spread shocks have been emphasized by Eggertsson and Woodford (2003), Christiano, Eichenbaum, and Rebelo (2011), Corsetti, Kuester, Meier, and Muller (2010) and Woodford (2011). These shocks are often assumed to follow a two-state Markov process in which the credit-spread shock disappears each period with a fixed probability, with aggregate output and inflation recovering as soon as the exogenous shock stops operating.

While this approach has been fruitful in suggesting suitable monetary and fiscal policy responses to such shocks, it has several somewhat unattractive features. It relies heavily on the persistence of a shock that evaporates according to an exogenous process, and recession ends as soon as the exogenous negative shock ends.² Furthermore, this approach does not do justice to an independent role for expectations.

¹For different arguments and explanations for long-lasting stagnation see, for example, Summers (2013), Teulings and Baldwin (2014), Eggertsson and Mehrotra (2014) and Benigno and Fornaro (2018).

²There is also an issue with existence of a rational expectations solution when the probability of the shock ending is too small. A related issue for calibrated models is the length of time that Japan has been at the ZLB.

A second approach, emphasized by Benhabib, Schmitt-Grohe, and Uribe (2001), focuses squarely on the existence of multiple rational expectations equilibria (REE) when the interest-rate rule is subject to the ZLB. In particular, in addition to the intended steady state at the inflation rate targeted by monetary policy, there is a second, unintended steady state at a low inflation or modest deflation rate, as well as perfect foresight paths converging to the unintended steady state. This multiplicity was emphasized in Bullard (2010). Figure 1 gives a scatter plot of core inflation vs. the policy interest rate, as originally done in Bullard (2010) for Japan and US data and extended by Honkapohja (2016) using also euro area data.

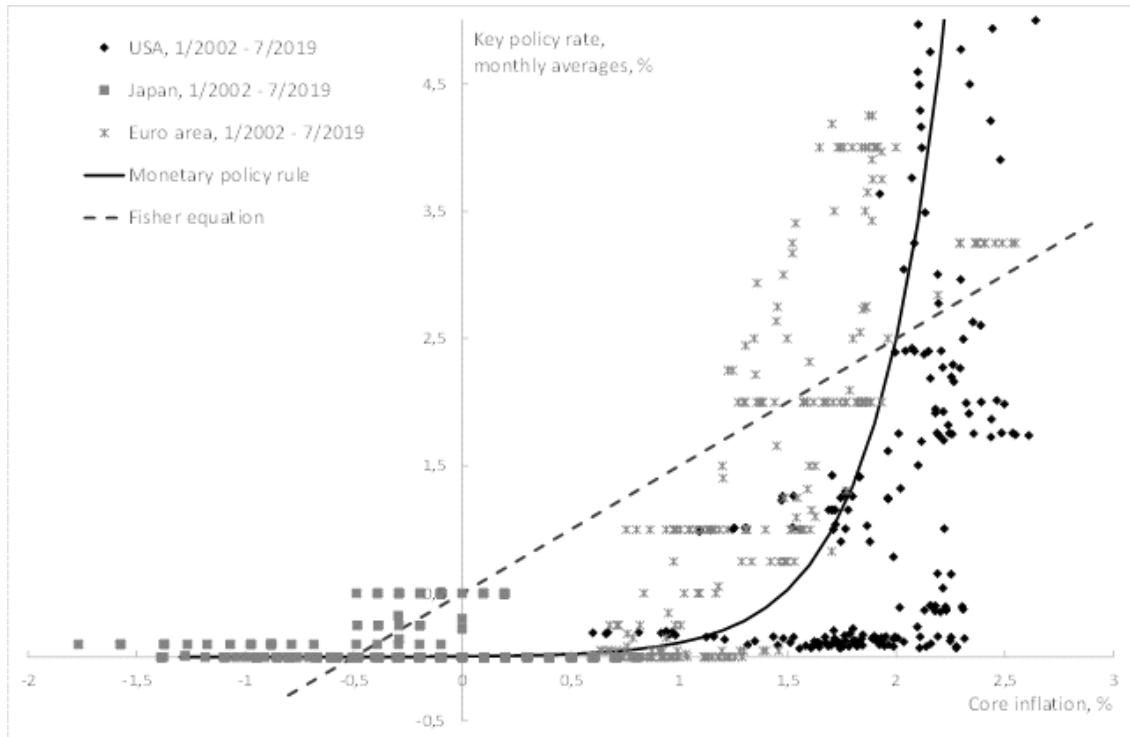


Figure 1: Interest rate vs inflation in Japan, US and euro area

Figure 1 uses monthly data, over 1/2002 to 7/2019 for euro area, US and Japan, and combines them in one figure.³ The illustrated policy rule is drawn with a two-percent inflation target and is merely used to provide a common reference since the two percent target does not exactly match either U.S. or euro area practice.

³See Appendix E for details of data used in Figures 1 and 2.

Inflation and interest rates at the two steady states in Figure 1 correspond to the two intersections of the Fisher equation and a Taylor-type interest rate rule. The Japanese data from this period is essentially entirely within the liquidity trap, while the US and euro area data show a mixture of liquidity-trap and non-liquidity trap periods. Both the US and the euro area had brief periods of deflation during 2009 and the Great Recession, followed by a period of inflation. However more recently, since 2013, inflation in both the euro area and the US has often been below target and sometimes shown signs of decline. Figure 1 thus suggests some possibility of convergence to an unintended low inflation steady state.

A major problem with this second approach is its neglect of the association of the ZLB with periods of recession, low output and stagnation. Although there is a long-run trade-off in the NK model between output and inflation, the extent of this trade-off is quite minor: at the unintended low inflation steady state the level of aggregate output is only very slightly below that of the intended steady state in Figure 1.

Figure 2, which gives real GDP per capita since 2001 for the US, Japan and the euro area, illustrates the association of depressed output levels in these countries with the ZLB. This is inconsistent with the view of two steady-states with nearly identical output levels in the second approach. Taken together with Figure 1, there appears also to be the possibility of stagnation, i.e. persistently depressed levels of output, at low inflation or deflation steady states. For the US, the decrease from 2007Q4 to 2009Q2 was about 6.0%. Given an underlying trend growth in the US of real GDP per capita of 2% per year, one would have expected 3% total growth over this period, so one could argue this corresponds to a 9% GDP gap. For Japan, the decrease in GDP per capita from 1997Q1 to 1999Q1 was 3.5% and from 2008Q1 to 2009Q2 was 7.5%. For the euro area the drop in GDP per capita from 2008Q1 to 2009Q2 was 5.5%. Again, allowing for usual trend growth in GDP per capita, the resulting GDP gaps would be larger.

Another objection to the two-steady state view of recent events is that the unintended low-inflation steady state is not stable under adaptive learning. This point was emphasized in Evans, Guse, and Honkapohja (2008) and Benhabib, Evans, and Honkapohja (2014). We expand on this at length below, but the key point is that this makes it implausible that the economy will converge to the unintended steady state.

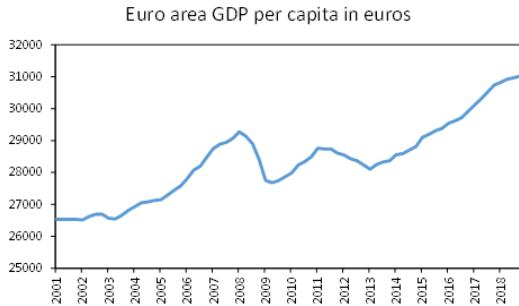


Figure 2A: GDP per capita in Euro area

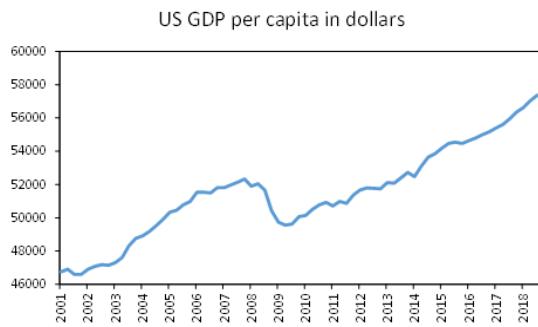


Figure 2B: GDP per capita in United States

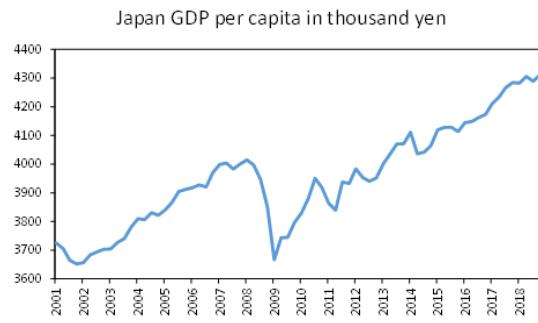


Figure 2C: GDP per capita in Japan

A third approach relies on sunspot equilibria that can also be shown to exist when there are two steady states. A sunspot is modelled as a two-state Markov process

with fixed transition probabilities. This can either be a stationary 2-state sunspot equilibrium, as in Aruoba, Cuba-Borda, and Schorfheide (2018) or a 2-state sunspot equilibrium with an absorbing state at the targeted steady state, as in Mertens and Ravn (2014). In this approach the state corresponding to deflation and recession is not due to a fundamental shock, but to a pure confidence shock.

This approach is attractive in that it gives full weight to the multiple equilibria issue. However it also has disadvantages. There is the practical question of exactly what variable is used to coordinate expectations, and there is again the issue of stability under learning. Two-state sunspot equilibria are not locally stable under learning when they are near two steady states, one of which is not locally stable under learning as in the present case; e.g. see Evans and Honkapohja (2001), Chapter 12.

There is also an issue concerning the relatively small magnitude of recessions on this approach. The size of recessions appears to be greatest in the case of a Markov sunspot equilibrium with an absorbing state. However, even in this case the size of the recession is relatively mild: in the illustrations given in Mertens and Ravn (2014) the impact on output is -1.6% . This is a magnitude well below those in the Great Recession, which in turn were relatively small compared to the Great Depression during which substantial deflation and the ZLB was also attained; real GDP figures for the US show a 26.5% drop between 1929 and 1933.

This discussion motivates the approach that we take in the current paper. The dynamics of private-sector expectations are modeled using the AL approach rather than standard RE. RE assumes a great deal of knowledge on the part of agents and also implicitly assumes coordination of agents on those expectations. These criticisms of RE are particularly forceful when the economy is in an unusual situation, i.e. outside the usual regime of positive inflation and interest rates. In such circumstances the government may also need to consider policies outside the usual range of experience.

In Evans, Guse, and Honkapohja (2008) and Benhabib, Evans, and Honkapohja (2014) AL was introduced into the NK model with two steady states arising from the ZLB. These papers showed that while the unintended steady state is not locally stable under learning, it is on the edge of a region in which inflation and output could gradually fall apparently without positive lower bound.⁴ In the current paper we extend the nonlinear NK model and show that there are natural lower bounds to output and inflation that generate a third, stagnation, steady state. These bounds are underpinned by the assumption, used for example by Christiano and Eichenbaum (1992), that government spending is a partial substitute for private consumption, and by the assumption that the government has priority to its chosen level of spending.

⁴This dynamic is sometimes called a deflation trap.

The associated stagnation steady state should be viewed as a theoretical limit, which acts an ultimate attractor within the stagnation region, preventing a return to the targeted steady state in the absence of aggressive policy. Our interest is in dynamics when the economy is in or near the stagnation region, and in the policies that can move the economy back towards convergence to the targeted steady state.

Formally, the three steady states all satisfy RE, and the model is therefore indeterminate. AL resolves the indeterminacy issue in the sense that, given initial expectations and the learning rule, the time path of the economy is pinned down. We show that while the usual targeted steady state is locally stable under learning, the unintended steady state emphasized by Benhabib, Schmitt-Grohe, and Uribe (2001) is unstable. With unchanged policy rules there will ultimately either be convergence to the intended steady state or expectations will be trapped in the stagnation regime.⁵

The starting point for our approach is that low output and inflation during the period of exogenous discount rate, credit or other shocks, may have made agents pessimistic about the future. These pessimistic output and inflation expectations may continue for a time after the exogenous shocks have ceased, and the economy may lie outside the domain of attraction of the targeted steady state.⁶ The dynamics can depend sensitively on initial conditions near the boundary of the domain of attraction. If expectations are too pessimistic the economy can become trapped in the stagnation regime under normal policy. If the economy has settled into a stagnation regime, can fiscal policy prevent stagnation and return the economy to the targeted steady state?

Earlier work has shown that monetary and fiscal policy effects under AL can be significantly different from those based on the RE assumption.⁷ The AL approach

⁵Our stability results bear some similarities to other macroeconomic learning models with multiple steady states, e.g. Marcer and Nicolini (2003) and Evans, Honkapohja, and Romer (1998). The former examines policies to avoid hyperinflation in seigniorage models of inflation. The latter demonstrates the possibility of self-fulfilling cycles between high and low growth rates.

⁶A similar point arises in connection with the large negative productivity shocks in 2020 due to the coronavirus pandemic. The course of the economy will of course depend heavily on the course of the “intrinsic” virus shocks (and a serious model would take into account, e.g., sectoral shifts in demand). However, even after these shocks have receded, there may well be a pessimistic expectational overhang of the type considered in this paper.

⁷The Great Recession and the ZLB have led to renewed interest in fiscal policy and a fairly voluminous recent literature; see, e.g., Ramey (2011), Leeper, Traum, and Walker (2011), Coenen et al. (2012), Woodford (2011) and Hagedorn, Manovskii, and Mitman (2019). In the US the 2009 American Recovery and Reinvestment Act provided an approximate \$800 billion fiscal stimulus over 2009-2011. In a 29 September 2019 Financial Times interview, Mario Draghi, the departing ECB president, said he had “talked about fiscal policy as a necessary complement to monetary policy since 2014. Now the need is more urgent than before.”

in the current paper is implemented as follows. We use and adapt the long-horizon anticipated-utility approach advocated by Preston (2005) and Eusepi and Preston (2010), extended for policy changes as discussed in Evans, Honkapohja, and Mitra (2009) and Mitra, Evans, and Honkapohja (2013). We implement this within a non-linear model that includes exogenous productivity and mark-up shocks, using natural bounded rationality assumptions to model decision-making. Agents are assumed to incorporate the announced path of future government spending and taxes into their intertemporal budget constraint, and thus take into account the known direct impact of the policy.⁸ However, agents are assumed not to know the general equilibrium effects of the temporary change in fiscal policy and use AL to forecast future output and inflation. Under AL agents update each period estimated coefficients in their forecast model, and the evolution of these parameters over time modulates the impact of fiscal policy under learning vis-a-vis RE.

The structure of our paper is as follows. In Section 2 we present the basic Rotemberg adjustment-cost version of the NK model with AL, extended to include partial substitutability between private and public consumption. Section 3 obtains the key existence and learning stability results for the different steady states, demonstrating in particular the possibility of a locally stable stagnation steady state and an associated stagnation region. Using E-stability analysis we characterize the global expectational dynamics under a simple learning rule and illustrate the different domains of attraction in a global phase diagram. We then show how to extend our framework to incorporate intrinsic stochastic shocks with learning rules updated over time using discounted least squares. Numerical simulations are used to illustrate the stochastic nature of the outcomes to varying degrees of pessimistic output expectations shocks.

In Section 4 we turn to policy, focusing on situations in which output expectations are sufficiently pessimistic that with high likelihood they would lead, under unchanged policy, to the economy becoming trapped in the stagnation regime. We focus on the impact of a temporary fiscal stimulus, of a stated magnitude and duration, and we provide numerical results for the success of this fiscal policy in moving the economy to a path converging to the targeted steady state. The impact of fiscal policy is highly nonlinear: for a given duration, a small stimulus may be unsuccessful, while a larger temporary stimulus can be effective in returning the economy to the targeted steady state. The effects are also stochastic, since convergence to the targeted steady state depends in part on the sequence of stochastic shocks. We find that the probability of success, i.e. avoiding stagnation, depends on the magnitude and length of fiscal stimulus.

Section 5 considers the implications of several important extensions. (i) Com-

⁸It would be possible to relax this assumption as in Mitra, Evans, and Honkapohja (2019).

bining expansionary fiscal policy with forward guidance in monetary policy can be beneficial as the percentage of success in avoiding stagnation is higher than if only fiscal policy is used. (ii) Policy delays reduce the efficacy of fiscal policy. (iii) A high discount rate and financial frictions can result in appearance of the stagnation regime, even at normal levels of output expectations and when inflation expectations are positive, if inflation expectations are significantly below target. (iv) A higher inflation target enlarges the domain of attraction of the targeted steady state. (v) The likelihood of stagnation is reduced if the inflation target has positive credibility.

Section 6 concludes with a discussion of the results for fiscal policy and the significance of the extensions.

2 The Model

2.1 Household-producers

The model is a generalization of the model in Benhabib, Evans, and Honkapohja (2014). There is a continuum of household producers $i \in [0, 1]$. The present value utility of agent i is subject to a standard flow budget constraint:

$$\text{Max } E_{0,i} \sum_{t=0}^{\infty} \beta^t U_{t,i} \left(c_{t,i} + \xi g_t, \frac{M_{t-1,i}}{P_t}, h_{t,i}, \frac{P_{t,i}}{P_{t-1,i}} - \pi^* \right) \quad (1)$$

$$\text{st. } c_{t,i} + m_{t,i} + b_{t,i} + \Upsilon_{t,i} = m_{t-1,i} \pi_t^{-1} + R_{t-1} \pi_t^{-1} b_{t-1,i} + \frac{P_{t,i}}{P_t} y_{t,i}, \quad (2)$$

where $c_{t,i}$ is the consumption aggregator consumed by i , $M_{t,i}$ and $m_{t,i} = M_{t,i}/P_t$ denote nominal and real money balances, $h_{t,i}$ is the labor input into production of good variety i and $b_{t,i}$ denotes the real quantity of risk-free one-period nominal bonds held by the agent i at the end of period t . g_t is government spending per capita which households treat as exogenous (see below for more detail). $\Upsilon_{t,i}$ is the lump-sum tax collected by the government, R_{t-1} is the nominal interest rate factor between periods $t-1$ and t , $P_{t,i}$ is the price of consumption good i , $y_{t,i}$ is output of good i , P_t is the aggregate price level (see below), and the inflation rate is $\pi_t = P_t/P_{t-1}$. Here π^* denotes the inflation rate targeted by policymakers.

It is assumed that utility functions are identical across agents. The subjective discount factor is denoted by β . The utility function has the parametric form

$$U_{t,i} = \log(c_{t,i} + \xi g_t) + \chi \log \left(\frac{M_{t-1,i}}{P_t} \right) - \frac{h_{t,i}^{1+\varepsilon}}{1+\varepsilon} - \Phi \left(\frac{P_{t,i}}{P_{t-1,i}} - \pi^* \right),$$

where $\varepsilon > 0$. For the most part we analyze the widely considered case $\varepsilon = 1$. The utility of consumption includes both private consumption $c_{t,i}$ and public consumption $g_t > 0$ of the goods aggregator. Under standard policy g_t is assumed to be exogenous and fixed, i.e. $g_t = \bar{g} > 0$. ξ is a relative weight parameter, where $0 < \xi \leq 1$. This utility formulation is used e.g. by Christiano and Eichenbaum (1992). Our benchmark calibration is $\xi = 0.4$.⁹ The final term $\Phi(\cdot)$ describes the (utility) cost of adjusting prices in the spirit of Rotemberg. Such costs arise if agent i changes prices at a different rate from the inflation target of the central bank π^* . We use the Rotemberg formulation rather than the Calvo model of price stickiness because it enables us to study global dynamics in the nonlinear system.

When needed, notation $c_{t,i}(j)$ is used when reference is made to consumption of variety j by agent i , i.e. the consumption aggregate for agent i is

$$c_{t,i} = \left[\int_0^1 c_{t,i}(j)^{(\nu_t-1)/\nu_t} dj \right]^{\nu_t/(\nu_t-1)}.$$

Note that below $y_{t,i}$ denotes production of good variety i , $P_{t,i}$ is price of variety i and $h_{t,i}$ is labor supply of household-producer i used in production of good variety i .

The household decision problem is also subject to the usual “no Ponzi game” (NPG) condition. In (1) the expectations $E_{0,i}(\cdot)$ are in general subjective and they may not be rational. This approach is called anticipated utility maximization over an infinite horizon (IH). We discuss this further below.

Production function for good variety i is given by

$$y_{t,i} = A_t h_{t,i}^\alpha,$$

where $0 < \alpha < 1$. Here A_t is a productivity shock to all production with mean $\bar{A} > 0$. The production functions including the shock are taken to be identical across agents. Output is differentiated and firms operate under monopolistic competition. Each household-firm faces a downward-sloping demand curve

$$P_{t,i} = \left(\frac{y_{t,i}}{y_t} \right)^{-1/\nu_t} P_t, \text{ where } P_t = \left[\int_0^1 P_{t,i}^{1-\nu_t} di \right]^{1/(1-\nu_t)}. \quad (3)$$

Here $P_{t,i}$ is the profit maximizing price set by firm i consistent with its production $y_{t,i}$. The parameter ν_t is the elasticity of substitution between two goods. $\nu_t > 1$ and is taken to be random and stationary. y_t is aggregate output. ν_t and y_t are

⁹Christiano and Eichenbaum (1992) do not calibrate ξ but contrast the implications of $\xi = 0$ and $\xi = 1$. Qualitatively our results are not sensitive to the value $0 < \xi < 1$.

exogenous to the firm.

The government's flow budget constraint in real terms is

$$b_t + m_t + \Upsilon_t = g_t + m_{t-1}\pi_t^{-1} + R_{t-1}\pi_t^{-1}b_{t-1}, \quad (4)$$

where g_t denotes government consumption of the aggregate good,¹⁰ b_t is the real quantity of government debt, and Υ_t is the real lump-sum tax collected. Households are assumed to be Ricardian in the sense that they expect the government's intertemporal budget constraint to be satisfied in expectation, i.e.,

$$\lim_{s \rightarrow \infty} \hat{E}_{t,i}(D_{t,t+s})^{-1}b_{t+s} = 0, \text{ where } D_{t,t+s} = \prod_{j=1}^s r_{t+j} \text{ with } r_{t+j} \equiv \frac{R_{t+j-1}}{\pi_{t+j}}. \quad (5)$$

Our general approach, in line with the AL literature, rests on the temporary equilibrium concept originally introduced by Hicks (1946) and the Stockholm school of economic thought. At each point in time t , exogenous random variables are realized, expectations of relevant future variables are formed by economic agents, and their optimal decision rules are formed conditional on those expectations. Imposing market clearing then determines the time t temporary equilibrium levels of all variables including aggregate output and inflation. We then move to period $t+1$, when new realizations of exogenous variables are obtained, expectations by agents are revised and a new temporary equilibrium is generated. Adaptive learning provides the key step of specifying how the forecast rules, used to form expectations conditional on the information available, are revised over time.

In order to make our assumptions transparent, we develop the model in terms of individual agent decision-making. However, we then impose the standard representative-agent assumption, which enables us to present our core results most simply.

2.1.1 Consumption function and aggregate output

The consumption Euler equation is

$$\begin{aligned} (c_{t,i} + \xi g_t)^{-1} &= \beta R_t \hat{E}_{t,i} (\pi_{t+1}^{-1} (c_{t+1,i} + \xi g_{t+1})^{-1}) \\ &= \beta \hat{E}_{t,i} (r_{t+1} (c_{t+1,i} + \xi g_{t+1})^{-1}), \end{aligned} \quad (6)$$

¹⁰ g_t is allocated to the different varieties in the same way as the allocation of any given private consumption. This permits consistent aggregation of market clearing in markets for each variety i to usual market clearing in aggregates. See Leith and Wren-Lewis (2013).

provided $c_{t,i} > 0$. The household's consumption decision rule is obtained by combining iterations of (6) with the household intertemporal budget constraint and its perceived intertemporal budget constraint for the government.¹¹ Appendix A shows that combining the latter two yields

$$c_{t,i} = \frac{P_{t,i}}{P_t} y_{t,i} - g_t + \sum_{s=1}^{\infty} \hat{E}_{t,i} (D_{t,t+s})^{-1} \left(\frac{P_{t+s,i}}{P_{t+s}} y_{t+s,i} - c_{t+s,i} - g_{t+s} \right). \quad (7)$$

At this point we impose our first bounded-rationality assumption. The subjective expectations in (6) and (7) are expectations of nonlinear functions of future random variables. Even if agents knew these joint probability distributions this would be a difficult calculation, and in addition, since these distributions are unknown, they would need to be estimated. We make the assumption, which we view as realistic, that agents deal with these issues by in effect assuming *point expectations*; that is, agents treat the expectation of a nonlinear function of random variables as equal to the nonlinear function of the expectations. Put differently, they act as if all the probability density of each random variable were concentrated at its expected value. The quality of this approximation depends, of course, on the severity of nonlinearities and the size of the shock variances. However we believe our assumption is natural, within a bounded rationality context, because it can be plausibly implemented by agents as an approximation to optimal decision-making.

Using the point expectations assumption, with expectations denoted by superscript e on each variable, iterations of (6) can be written

$$c_{t+s,i}^e = -\xi g_{t+s}^e + \beta^s (D_{t,t+s}^e) (c_{t,i} + \xi g_t). \quad (8)$$

and substitution into (7) yields the consumption function

$$c_{t,i} = \max \left\{ 0, (1 - \beta) \left[\frac{P_{t,i}}{P_t} y_{t,i} - g_t \left(1 + \frac{\xi \beta}{1 - \beta} \right) \right] + (1 - \beta) \sum_{s=1}^{\infty} (D_{t,t+s,i}^e)^{-1} \left[\left(\frac{P_{t+s,i}}{P_{t+s}} \right)^e y_{t+s,i}^e - g_{t+s,i}^e (1 - \xi) \right] \right\}. \quad (9)$$

Here the non-negativity constraint for consumption is made explicit. We treat the consumption function as a decision rule that depends on forecasts of future incomes $\left(\frac{P_{t+s,i}}{P_{t+s}} \right)^e y_{t+s,i}^e$, taxes $g_{t+s,i}^e$ and the discount factors $D_{t,t+s,i}^e$. Further details of (9) are

¹¹Non-Ricardian households could also be considered. See Benhabib, Evans, and Honkapohja (2014).

discussed below.

Market clearing in aggregate and for each goods variety

$$y_t = \int_0^1 (c_{t,i} + g_t) di \text{ and } y_{t,i} = \int_0^1 (c_{t,j}(i) + g_t(i)) dj$$

hold in the usual way. It is assumed the government can require that its demand is always met, so we have

$$\begin{aligned} y_{t,i} &\geq g_t(i) \text{ for all } i \\ y_t &\geq g_t. \end{aligned}$$

Government purchases are distributed equally to the households. One interpretation is that government guarantees a subsistence level of consumption to households and agents are required to pay their taxes and hence must work to produce at least the amounts that government purchases.

2.1.2 Production decisions

The adjustment cost function $\Phi\left(\frac{P_{t,j}}{P_{t-1,j}}\right)$ is assumed to be asymmetric in $P_{t,j}/P_{t-1,j}$, and we use the Linex function. For its functional form see e.g. Kim and Ruge-Murcia (2009).

$$\Phi\left(\frac{P_{t,i}}{P_{t-1,i}}\right) \equiv \phi \left[\frac{\exp(-\psi(P_{t,i}/P_{t-1,i} - \pi^*)) + \psi(P_{t,i}/P_{t-1,i} - \pi^*) - 1}{\psi^2} \right],$$

where $\phi > 0$ and we assume the case $\psi > 0$. The first-order condition for optimal price setting is

$$\begin{aligned} 0 &= \frac{\partial U_{t,i}}{\partial P_{t,i}} + \beta E_{t,i} \frac{\partial U_{t+1,i}}{\partial P_{t,i}} = \frac{\nu_t}{\alpha} h_{t,i}^{\varepsilon+1} \frac{1}{P_{t,i}} - \Phi'(\pi_{t,i}) \frac{1}{P_{t-1,i}} \\ &\quad + (c_{t,i} + \xi g_t)^{-1} (1 - \nu_t) y_t \left(\frac{P_{t,i}}{P_t} \right)^{-\nu_t} \frac{1}{P_t} + \beta \Phi'(\pi_{t+1,i}^e) \left(\frac{P_{t+1,i}}{P_{t,i}^2} \right)^e, \end{aligned}$$

where again we have used point expectations and here $\pi_{t,i} = P_{t,i}/P_{t-1,i}$. This form is a modification of the FOC in Benhabib, Evans, and Honkapohja (2014) (appendix)¹² with different utility and adjustment cost functions. Multiplying by $P_{t,i}$ and using

¹²The first and second terms in the expression for $\frac{\partial U_{t,i}}{\partial P_{t,i}}$ have the wrong signs in Benhabib et al., p. 236. This typo does not affect their subsequent calculations.

$y_{t,i} = A_t h_{t,i}^\alpha$ the FOC can be written as

$$\Phi'(\pi_{t,i})\pi_{t,i} = \frac{\nu_t}{\alpha} \left(\frac{y_{t,i}}{A_t} \right)^{(1+\varepsilon)/\alpha} + (c_{t,i} + \xi g_t)^{-1} (1 - \nu_t) y_t \left(\frac{P_{t,i}}{P_t} \right)^{1-\nu_t} + \beta \Phi'(\pi_{t+1,i}^e) \pi_{t+1,i}^e.$$

Appendix A shows that the function

$$\Phi'(\pi)\pi = \frac{\phi}{\psi} \pi (-\exp(-\psi(\pi - \pi^*)) + 1)$$

is monotonically increasing above a critical value $\check{\pi}$, given by the condition $\frac{d}{d\pi}\Phi'(\pi)\pi = \frac{\phi}{\psi}(1 - (1 - \phi\pi)\exp(-\psi(\pi - \pi^*))) = 0$, and $\check{\pi}$ is increasing in ψ with $\lim_{\psi \rightarrow \infty} \check{\pi} = \phi^{-1}$ and (ii) $\check{\pi}$ is decreasing in ϕ with $\lim_{\phi \rightarrow \infty} \check{\pi} = 0$ *ceteris paribus*. Throughout the paper we restrict attention to regions for which $\pi > \check{\pi}$.

Let

$$\zeta_{t,i} = \frac{\nu_t}{\alpha} \left(\frac{y_{t,i}}{A_t} \right)^{(1+\varepsilon)/\alpha} - (\nu_t - 1) (c_{t,i} + \xi g_t)^{-1} y_t \left(\frac{P_{t,i}}{P_t} \right)^{1-\nu_t}. \quad (10)$$

Here

$$\frac{y_{t,i}}{A_t} = \frac{\int_0^1 c_{t,j}(i) dj + g_t(i)}{A_t} = \frac{c_t(i) + g_t(i)}{A_t}$$

is the total demand for variety i .

Note that in (10) the term $y_t \left(\frac{P_{t,i}}{P_t} \right)^{1-\nu_t}$ combines y_t , which is exogenous to the firm, with the relative price $\frac{P_{t,i}}{P_t}$, in which the aggregate price level is exogenous while $P_{t,i}$ is a decision variable of the firm. Iterating the FOC forward and using (10) we get

$$\Phi'(\pi_{t,i})\pi_{t,i} = \zeta_{t,i} + \sum_{s=1}^{\infty} \beta^s \zeta_{t+s,i}^e, \quad (11)$$

which we will treat as the infinite-horizon pricing decision rule, provided a transversality condition holds (which is a necessary condition for optimal price setting as in Benhabib, Evans, and Honkapohja (2014)). Here $\zeta_{t+s,i}^e$ is the point expectation of

$$\zeta_{t+s,i} = \frac{\nu_{t+s}}{\alpha} \left(\frac{y_{t+s,i}}{A_{t+s}} \right)^{(1+\varepsilon)/\alpha} - (\nu_{t+s} - 1) y_{t+s} \left(\frac{P_{t+s,i}}{P_{t+s}} \right)^{1-\nu_{t+s}} \times (c_{t+s,i} + \xi g_{t+s})^{-1},$$

where $y_{t+s,i} = c_{t+s}(i) + g_{t+s}(i)$ is the future market demand for variety i .

2.2 Boundedly-optimal decision-making

For both the consumption decision (9) and the pricing decision (11) our agents need to make forecasts of various future variables, and to proceed further we need to make additional bounded-rationality assumptions.

Consider, first, the pricing decision. To forecast $\zeta_{t+s,i}^e$ agent i needs to make forecasts of paths for the exogenous variables ν_{t+s} , A_{t+s} , the fiscal policy variable g_{t+s} , aggregate output y_{t+s} , relative price $\frac{P_{t+s,i}}{P_{t+s}}$, market demand $y_{t,i}$ for variety i and the marginal utility term in $c_{t+s,i} + \xi g_{t+s}$. As noted above, $\frac{P_{t+s,i}}{P_{t+s}}$ includes a future decision variable of the firm i . Thus formally (11) is a conditional decision-rule. Our approach is to assume that agents use this conditional decision rule supplemented by forecasts of the future relative prices $\frac{P_{t+s,i}}{P_{t+s}}$ that they themselves will be setting for their varieties. Thus we share with Eusepi and Preston (2010) the assumption that agents are infinite-horizon anticipated-utility optimizers, but in contrast to them the agents do not assume that their future pricing decisions $P_{t+s,i}$ will be consistent with what would be their optimal choices under current expectations of the variables exogenous to their decision-making, including future aggregate inflation and aggregate output.¹³ Instead we assume that agents use adaptive learning based on observed data on $\Xi_{t,i} = P_{t,i}/P_t$ to forecast their expected future relative price.

Similarly, if heterogeneous agents were allowed for, each agent would need to forecast its own output (demand) $y_{t+s,i}$ as well as aggregate output y_{t+s} . In the representative agent case these are, of course, identical, and we will assume that agents have learned this relationship. Consequently the marginal utility term in $c_{t+s,i} + \xi g_{t+s}$ can be conveniently forecasted using the market clearing condition which yields

$$c_{t+s,i} + \xi g_{t+s} = y_{t+s,i} - (1 - \xi)g_{t+s} = y_{t+s} - (1 - \xi)g_{t+s}.$$

In our discussion of fiscal policy we focus on the case in which the path of future government spending is credibly announced and is therefore known. Hence $c_{t+s,i} + \xi g_{t+s}$ can be forecasted using forecasts of aggregate output and the announced path of government spending.

The same points just discussed apply to the representative agent's consumption decision (9). The term $\left(\frac{P_{t+s,i}}{P_{t+s}}\right)^e$ will be replaced by a forecast of relative price $\Xi_{t,i}$ and $y_{t+s,i}^e$ will be replaced by y_{t+s}^e . We can now summarize the key model equations for the temporary equilibrium at t given agents' expectations.

¹³Note that the agent's forecasts of future aggregate variables *will* in general be revised over time as new data become available.

2.3 Temporary equilibrium with representative agents

From the beginning it has been assumed the utility and production functions of agents are identical, to which the assumption of identical expectations is now made explicit. In the representative agent economy with identical agents and expectations we have $c_{t,i} = c_t$, $y_{t,i} = y_t$ and also $P_{t,i} = P_{t,j} = P_t$ for all i, j in the current and previous periods. As just discussed in the previous section we allow for expectations $P_{t+s,i}^e$ to differ from P_{t+s}^e . The representative agent assumption nonetheless implies that

$$\frac{P_{t,i}}{P_t} y_{t,i} = y_t \text{ and } \pi_{t,i} = \pi_t$$

in temporary equilibrium, and the inflation factor is given by

$$\Phi'(\pi_t) \pi_t = \zeta_t + \sum_{s=1}^{\infty} \beta^s \zeta_{t+s}^e, \quad (12)$$

where

$$\begin{aligned} \zeta_t &= \frac{\nu_t}{\alpha} \left(\frac{y_t}{A_t} \right)^{(1+\varepsilon)/\alpha} - (\nu_t - 1) y_t \times (c_t + \xi g_t)^{-1} \text{ and where} \\ \zeta_{t+s}^e &= \frac{\nu_{t+s}^e}{\alpha} \left(\frac{y_{t+s}^e}{A_{t+s}^e} \right)^{(1+\varepsilon)/\alpha} - (\nu_{t+s}^e - 1) y_{t+s}^e (\Xi_t^e)^{1-\nu_{t+s}^e} \times (y_{t+s}^e - (1 - \xi) g_{t+s}^e)^{-1} \end{aligned} \quad (13)$$

is the point expectation for all agents and varieties i .

For the representative agent model the consumption function is

$$\begin{aligned} c_t &= \max \left\{ 0, (1 - \beta) \left[y_t - g_t \left(1 + \frac{\xi \beta}{1 - \beta} \right) \right] + \right. \\ &\quad \left. (1 - \beta) \sum_{s=1}^{\infty} (D_{t,t+s}^e)^{-1} (\Xi_t^e y_{t+s}^e - g_{t+s}^e (1 - \xi)) \right\} \end{aligned} \quad (14)$$

where $\Xi_t^e y_{t,i} = y_t$, expected gross income is $\Xi_t^e y_{t+s}^e$, and $D_{t,t+s}^e = \prod_{j=1}^s r_{t+j}^e$, where $r_{t+j} \equiv \frac{R_{t+j-1}}{\pi_{t+j}}$. In addition market clearing requires

$$y_t = c_t + g_t. \quad (15)$$

To complete the temporary equilibrium equations we specify monetary policy as

a forward-looking interest rate rule given by¹⁴

$$R_t = R(\pi_{t+1}^e, y_{t+1}^e) = 1 + (R^* - 1) \left(\frac{\pi_{t+1}^e}{\pi^*} \right)^{BR^*/(R^*-1)} \left(\frac{y_{t+1}^e}{y^*} \right)^{\phi_y}, \quad (16)$$

where $B > 1$ and $\phi_y \geq 0$. Here $\pi_t = P_t/P_{t-1}$ is the inflation factor, $\pi^* \geq 1$ is the inflation target, $R^* = \beta^{-1}\pi^*$ and y^* is the target level of output, which we assume is equal to the output level at the nonstochastic targeted steady state. Note that $R_t \geq 1$, i.e. this interest-rate rule satisfies the zero lower bound for net interest rates. For simplicity agents are assumed to know the interest rate rule. Hence

$$r_{t+j}^e \equiv \frac{R(\pi_{t+j}^e, y_{t+j}^e)}{\pi_{t+j}^e}$$

and the discount factor is

$$D_{t,t+s}^e = \prod_{j=1}^s r_{t+j}^e, \text{ where } r_{t+j}^e \equiv \frac{R(\pi_{t+j}^e, y_{t+j}^e)}{\pi_{t+j}^e}. \quad (17)$$

We collect the expectation variables, which are taken as given in the time t equilibrium: exogenous markup shocks $\{\nu_{t+s}^e\}$ and productivity shocks $\{A_{t+s}^e\}$, relative price and output variables $\{\Xi_t^e\}$ and $\{y_{t+s}^e\}$, government spending $\{g_{t+s}^e\}$, and inflation $\{\pi_{t+s}^e\}$, as well as the implied discount factors $D_{t,t+s}^e$. In the representative agent model, given these expectations, temporary equilibrium consumption, output, interest rates and inflation are given by (12), (13), (14), (15) and (16).

3 Dynamics of the Economy

We now turn to how expectations are revised over time and to the dynamics under adaptive learning. Although we develop our approach within a stochastic setting, it is illuminating to begin with the non-stochastic case, in which adaptive learning rules are particularly simple.

¹⁴We have also considered contemporaneous rules $R_t = R(\pi_t, y_t)$ with a similar functional form, and the main results appear to be unchanged. The forward-looking rule (16) is both formally and computationally simpler to implement. The most straightforward interpretation of (16) is that the central bank policy rate reacts to private-sector expectations.

3.1 Steady states and learning in the nonstochastic case

Because the model is nonlinear and stochastic, we look to the nonstochastic version of the model to provide initial formal results and intuition, within which we can analyze “steady state learning” to get a global picture of the dynamics of the economy. If the random shocks are small, the nonstochastic version gives an approximation for the mean dynamics of the model. The shocks are fixed to be constants $\nu_t = \nu > 1$ and $A_t = A$ and also government spending and its forecasts are fixed and constant $g_{t+s} = g_{t+s}^e = \bar{g}$.

As foreshadowed, our final key bounded-rationality assumption is that forecasts of future variables are made using the adaptive learning approach. More specifically, in the stochastic case formulated below, we use the recursive least-square learning approach to expectation formation as developed in Bray and Savin (1986), Marcet and Sargent (1989) and Evans and Honkapohja (2001). On this approach agents forecast like econometricians, regressing variables to be forecasted on observed explanatory variable, updating the forecast rule coefficients as new data become available.

In the nonstochastic case agents’ forecasting model reflects a steady state and agents’ beliefs are about the long-run averages. Introducing the notation

$$(P_{t+s,i}/P_{t+s})^e = (P_{t,i}/P_t)^e \equiv \Xi_t^e, \quad y_{t+s}^e = y_t^e, \quad \text{and } \pi_{t+s}^e = \pi_t^e \text{ for all } s > 0,$$

adaptive learning rules for the non-stochastic case take the simple form

$$\Xi_t^e = \Xi_{t-1}^e + \omega(1 - \Xi_{t-1}^e), \quad (18)$$

$$y_t^e = y_{t-1}^e + \omega(y_{t-1} - y_{t-1}^e) \text{ and} \quad (19)$$

$$\pi_t^e = \pi_{t-1}^e + \omega(\pi_{t-1} - \pi_{t-1}^e), \quad (20)$$

where $0 < \omega < 1$ is the learning “gain” parameter. Adaptive learning usually focuses on cases with ω small and examines local stability of steady states for all sufficiently small $\omega > 0$. Adaptive-learning rules of the form (18)-(20) are often called “steady-state” learning.

Note that due to the representative agent economy assumption, temporary equilibrium implies $\Xi_t = 1$ for all t . For the discount factor we have

$$D_{t,t+s}^e = \left[\frac{R(\pi_t^e, y_t^e)}{\pi_t^e} \right]^s.$$

The behavioral functions are now:

(i) aggregate demand

$$y_t = \bar{g}(1 - \xi) + (\beta^{-1} - 1)(\Xi_t^e y_t^e - \bar{g}(1 - \xi)) \left(\frac{\pi_t^e}{R(\pi_t^e, y_t^e) - \pi_t^e} \right)$$

for $y_t \geq \bar{g}$.¹⁵

(ii) Phillips curve

$$\Phi'(\pi_t)\pi_t = \zeta_t + \sum_{s=1}^{\infty} \beta^s \zeta_{t+s}^e,$$

and where $c_t + \bar{g} = y_t$. Here

$$\begin{aligned} \zeta_t &= \frac{\nu}{\alpha} (y_t/A)^{(1+\varepsilon)/\alpha} - (\nu - 1) y_t \times (y_t - (1 - \xi)\bar{g})^{-1}, \\ \zeta_t^e &= \frac{\nu}{\alpha} (y_t^e/A)^{(1+\varepsilon)/\alpha} - (\nu - 1) \Xi_t^e y_t^e \times (y_t^e - (1 - \xi)\bar{g})^{-1}. \end{aligned}$$

Thus

$$\sum_{s=1}^{\infty} \beta^s \zeta_{t+s}^e = \left[\frac{\nu}{\alpha} (y_t^e)^{(1+\varepsilon)/\alpha} - (\nu - 1) \Xi_t^e y_t^e \times (y_t^e - (1 - \xi)\bar{g})^{-1} \right] \frac{\beta}{1 - \beta}$$

and the Phillips curve is given by

$$\begin{aligned} \Phi'(\pi_t)\pi_t &= \frac{\nu}{\alpha} (y_t/A)^{(1+\varepsilon)/\alpha} - (\nu - 1) y_t \times (y_t - (1 - \xi)\bar{g})^{-1} + \\ &\quad \frac{\beta}{1 - \beta} \left[\frac{\nu}{\alpha} (y_t^e/A)^{(1+\varepsilon)/\alpha} - (\nu - 1) \Xi_t^e y_t^e \times (y_t^e - (1 - \xi)\bar{g})^{-1} \right]. \end{aligned}$$

In a perfect-foresight steady state $y_t = y^e = y$ and $\pi_t = \pi_t^e = \pi$, we have

$$(1 - \beta)\Phi'(\pi)\pi = \frac{\nu}{\alpha} (y/A)^{(1+\varepsilon)/\alpha} - (\nu - 1) y \times (y - (1 - \xi)\bar{g})^{-1} \quad (21)$$

since $\Xi_t^e = \Xi_t = 1$. The Fisher equation and the interest rate rule are the remaining steady state equations. As is well known, there is a targeted steady state with $\pi = \pi^*$ and the level of output y^* determined from (21) with $\pi = \pi^*$. As is also well known, requiring $y > \bar{g}$ and $R(\pi, y)/\pi = \beta^{-1}$ with $R'(\pi, y) < \beta^{-1}$ results in a second steady state (π_L, y_L) with π_L and y_L determined from equations $R(\pi, y)/\pi = \beta^{-1}$ and (21) evaluated at (π_L, y_L) . We remark that if the ZLB were binding at $\pi = \pi_L$, so that

¹⁵We remark that for a range of values of π_t^e, y_t^e it is possible that $R(\pi_t^e, y_t^e) < \pi_t^e$. Below we deal with this issue which does not arise for local stability under steady state learning.

$R = 1$, then we would have $\pi_L = \beta$, i.e. there would be a net deflation rate of $1 - \beta$. Under our calibration of (16) $1 > \pi_L > \beta$ with $\pi_L \approx \beta$.

Finally, if output is constrained to the lower bound $y = g = \bar{g}$, then there exists a third, stagnation, steady state, with inflation π_S (actually deflation) at this steady state determined from (21) with $y_S = \bar{g}$, i.e.

$$(1 - \beta)\Phi'(\pi_S)\pi_S = \frac{\nu}{\alpha}(\bar{g}/A)^{(1+\varepsilon)/\alpha} - (\nu - 1)\xi^{-1}.$$

The condition for existence of the stagnation steady state is

$$\bar{g}/A < \left(\frac{\alpha(\nu - 1)}{\nu\xi} \right)^{\alpha/(1+\varepsilon)}$$

as $\Phi'(\pi)\pi < 0$ if and only if $\pi < \pi^*$. For the calibration below, the condition $\bar{g} < 1.338$ approximately is required.

We now turn to stability of the three steady states π^*, π^L and π_S under adaptive learning. E-stability, defined in terms of the ordinary differential equation (ODE) given below, is known to be the condition for local convergence of steady state learning to a (steady-state) fixed point. Here there are three expectations variables. Based on the above, temporary equilibrium is defined by the following equations for given expectations $(\pi_t^e, \Xi_t^e, y_t^e)$:

- Inflation

$$\begin{aligned} \pi_t &= Q^{-1} \left[\frac{\nu}{\alpha}(y_t/A)^{(1+\varepsilon)/\alpha} - (\nu - 1)y_t \times (y_t - (1 - \xi)\bar{g})^{-1} + \right. \\ &\quad \left. \frac{\beta}{1 - \beta} \left[\frac{\nu}{\alpha}(y_t^e/A)^{(1+\varepsilon)/\alpha} - (\nu - 1)\Xi_t^e y_t^e \times (y_t^e - (1 - \xi)\bar{g})^{-1} \right] \right] \\ &\equiv \tilde{G}_1(y_t, \Xi_t^e, y_t^e). \end{aligned}$$

where $Q(\pi_t) \equiv \Phi'(\pi_t)\pi_t$.

- Relative price

$$\Xi_t = 1.$$

- Aggregate output

$$\begin{aligned} y_t &= \max \left[\bar{g}, \bar{g}(1 - \xi) + (\beta^{-1} - 1) \left[(\Xi_t^e y_t^e - \bar{g}(1 - \xi)) \left(\frac{\pi_t^e}{R(\pi_t^e, y_t^e) - \pi_t^e} \right) \right] \right] \\ &\equiv \tilde{G}_2(\pi_t^e, \Xi_t^e, y_t^e). \end{aligned}$$

In general, for the vector of learning parameters θ , the E-stability ODE is $d\theta/d\tau = T(\theta) - \theta$, where $T(\theta)$ gives the corresponding actual temporary equilibrium outcome parameters for given perceived law of motion parameters θ . Here τ denotes “notional” time, which can, however be linked to real time t . From the above temporary equilibrium equations we obtain the E-stability differential equations:

$$\frac{d\Xi^e}{d\tau} = 1 - \Xi^e \quad (22)$$

$$\frac{dy^e}{d\tau} = F_y(\pi^e, \Xi^e, y^e) \equiv \tilde{G}_2(\pi^e, \Xi^e, y^e) - y^e \quad (23)$$

and

$$\frac{d\pi^e}{d\tau} = F_\pi(\pi^e, \Xi^e, y^e) \equiv \tilde{G}_1(\tilde{G}_2(\pi^e, \Xi^e, y^e), \Xi^e, y^e) - \pi^e. \quad (24)$$

We have the following stability and instability results:¹⁶

Proposition 1. (a) (i) *The targeted steady state at (π^*, y^*) is E-stable provided ϕ_y is not too large.* (ii) *The steady state (π_L, y_L) is not E-stable if ϕ_y is not too large.* (iii) *The steady state (π_S, y_S) is E-stable. Hence, provided ϕ_y is not too large,* (b) *for all $\omega > 0$ sufficiently small, under the learning rule (18)-(20), the steady state (π_L, y_L) is not locally stable and the steady states (π^*, y^*) and (π_S, y_S) are locally stable.*

We remark that the condition that ϕ_y not be too large is standard and known to be necessary, with forward-looking interest rate rules, in order to avoid indeterminacy of the targeted steady state.

Because we have fully specified the temporary equilibrium of the nonlinear system, we can extend our analysis numerically to look at the global system under learning.

3.2 Global analysis of E-stability dynamics

At this stage the simplifying assumption that $\Xi_{t+s}^e = 1$ holds for all s is made.¹⁷ Given the above local results, it is seen that this assumption is just a useful simplification enabling a two-dimensional analysis in expectation variables (π_t^e, y_t^e) . The model now consists of two key equations:

¹⁶See Appendix B for the proofs of Proposition 1, parts (i) - (iii).

¹⁷This assumption is made in earlier papers such as Benhabib, Evans, and Honkapohja (2014). In the representative-agent case, since $\Xi_t = 1$, all t , we necessarily have $\Xi_t \rightarrow 1$.

- Inflation: $\pi_t = G_1(y_t, y_t^e)$ where $G_1(y_t, y_t^e) \equiv \tilde{G}_1(y_t, 1, y_t^e)$
- Aggregate output: $y_t = G_2(\pi_t^e, y_t^e)$ where $G_2(\pi_t^e, y_t^e) \equiv \tilde{G}_2(\pi_t^e, 1, y_t^e)$.

As already noted, household producers are required to work and produce at least the quantity $g_t = \bar{g}$ in the current period. There is thus a low boundary for low levels of π_t^e at which private consumption is zero but positive $g_t = \bar{g}$ instituted by the government providing subsistence consumption. Interestingly, above the targeted steady state the bound $y_t \geq \bar{g}$ is also binding for sufficiently high values of π_t^e . This is because the real interest rate $R(\pi_t^e, y_t^e)/\pi_t^e$ then becomes very high (due to the Taylor rule) reducing c_t to zero.¹⁸

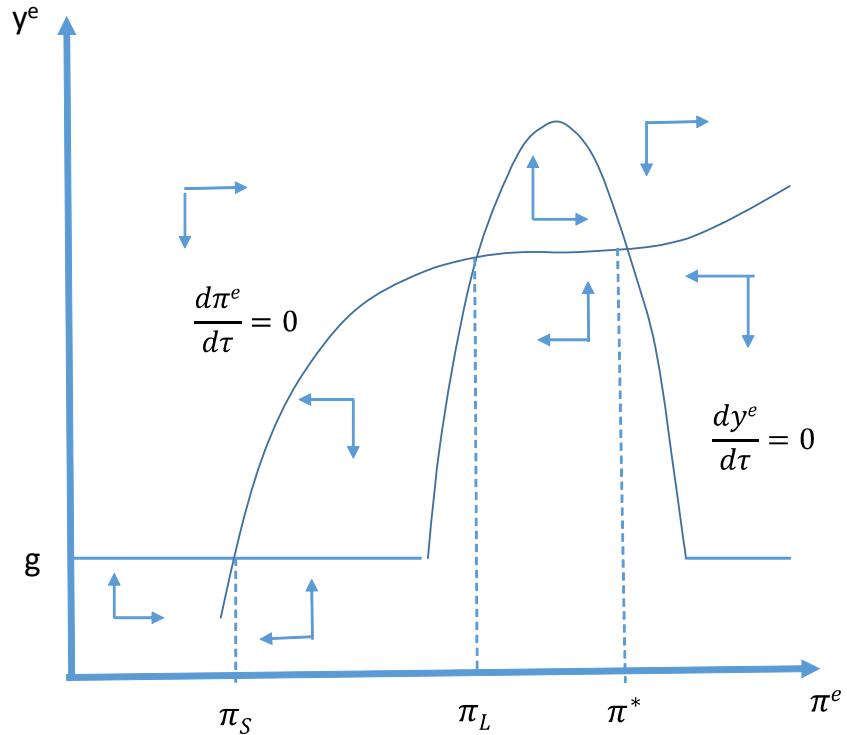


Figure 3: Global E-stability dynamics.

We begin by illustrating the global E-stability dynamics in the (π^e, y^e) space. It is assumed that government fiscal policy is constant, so that $g_t = g_{t+s}^e = \bar{g}$ for this

¹⁸In Figure 4 this phenomenon would appear in the curve $dy^e/d\tau = 0$ which gradually turns near-horizontal large $\pi^e > \pi^*$.

part of the analysis. In later sections fiscal policy in the form of changes in g_t are analyzed. The following numerical parameter values are used when constructing the next figures. $\pi^* = 1.005$, $\beta = 0.99$, $\alpha = 0.7$, $\xi = 0.4$, $\bar{A} = 1.13$, $\nu = 13.5$, $\phi = 75$, $\psi = 20$, $\varepsilon = 1$, $\bar{g} = 0.2$, $B = 1.5/R^*$, $\phi_y = 8.25$. Most of these are typical parameter values – see the comments on the calibration in the Appendix D.¹⁹ We choose the values $\phi = 75$, $\psi = 20$ based on comparing Linex-type functions to a quadratic adjustment cost function at the most common range for π .

With this numerical specification we construct the phase diagram of E-stability dynamics (23) - (24), under steady-state learning and with $\Xi^e \equiv 1$. Figure 3 provides a sketch of the global E-stability dynamics that includes all three steady states: target steady state (π^*, y^*) , liquidity trap steady state (π_L, y_L) and the stagnation steady state (π_S, \bar{g}) .

The two steady states (π^*, y^*) and (π_L, y_L) have been widely discussed in the literature. These are the two steady states seen also in Figure 1. Figure 3 illustrates that the steady state at π^* is locally stable under the learning dynamics, while the one at π_L is locally unstable. These observations are well known, see e.g. Benhabib, Evans, and Honkapohja (2014). At the stagnation steady state output $y = g$ is at the minimal level, with households receiving only g as subsistence consumption (private consumption is zero). This steady state also involves rapid deflation. Formally, (π_S, g) is locally stable, and more specifically is a sink with dynamics nearby that are not oscillatory.

Noting the saddle-point nature of the unstable middle steady state (π_L, y_L) in Figure 3 it is possible to construct the domain of attraction for the locally stable targeted steady state under the E-stability dynamics.²⁰ The domain of attraction of the targeted steady state, shown by the top panel in Figure 4, is the “liver-shaped” region with a narrow tail toward the north-west. For any expectations (π_t^e, y_t^e) inside this domain of attraction,²¹ in the non-stochastic case under consideration, the economy will converge under learning to the targeted steady state, whereas it will diverge to the stagnation steady state from all points outside this domain. In other words, under imperfect knowledge, there is a real possibility that after significant shocks, leading to an adverse shift in expectations (π^e, y^e) , the economy can move into, and become stuck in, a region leading to stagnation under unchanged policy,

¹⁹The calibration for ϕ_y is in Appendix F.

²⁰We note that in global E-stability dynamics there can be a problem that $R(\pi^e, y^e) < \pi^e$ for some configurations (π^e, y^e) . This issue does not arise in Figure 4, but would arise if $y^e < 0.9$ and $\pi^e \approx 1$. We address this point in Section 3.3 in the context of global numerical simulations.

²¹For brevity, going forward we often use “domain of attraction” to refer to the domain of attraction of the targeted steady state.

i.e. if government spending remains constant and the central bank leaves unchanged its interest-rate rule.²² It is convenient to refer to the part of the domain of attraction of the stagnation steady state in which $\pi^e < \pi_L$ and $d\pi^e/d\tau < 0$ as the stagnation “regime” or “region” or as the “stagnation trap.”²³ For future reference, when $\pi^e = \pi^*$ the lower boundary of the domain of attraction is approximately $y^e = 0.98792$.

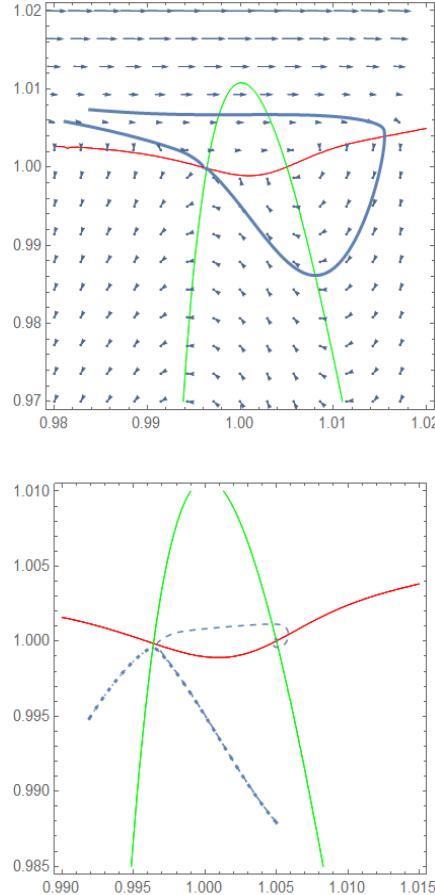


Figure 4: The top panel shows the domain of attraction of the targeted steady state while the bottom panel shows time paths to stagnation or to targeted steady state. π^e is on horizontal and y^e on vertical axis.

²²Parameter values of the monetary policy rule matter for the size of the domain of attraction. For example for $\phi_y = 0$ the domain of attraction is smaller than in our base case.

²³The term “deflation trap” is also used in the literature.

The bottom panel in Figure 4 illustrates time paths of the economy from a starting point at $\pi_0^e = \pi^*$ and y_0^e slightly below or slightly above the boundary of domain of attraction $y^e = 0.98792$. The dotted curve shows the time path of the economy from an initial value y_0^e slightly below 0.98792 while the dashed time path corresponds to y_0^e slightly above 0.98792. The two time paths are very close to each other until they get near the middle steady state (π_L, y_L) . They then evolve in very different ways: one path moving deep into the stagnation region, and the other path eventually converging to the targeted steady state in dampening oscillations. This extreme sensitivity to initial conditions is local to the boundary of the domain of attraction, and is a reflection of our global nonlinear temporary equilibrium set-up. However, this sensitivity occurs in a critical area and complicates decision-making for policymakers.

Figure 4 illustrates some challenges in the design of fiscal and monetary policy. From the top panel in Figure 4, it is evident that if the economy is within the stagnation region, e.g. just southwest of the lower arm of the domain of attraction of (π^*, y^*) , then sufficiently aggressive policy needs to be taken so that dynamics are transferred to inside the domain. Clearly the size of the policy change required will depend on the initial position (π^e, y^e) following the shock, and thus choosing the magnitude of the policy can be delicate.

Second, the bottom panel of Figure 4 shows that the model has sensitive dependence on initial conditions in a relevant area of the state space. The figure illustrates how, following an initial expectation shock, it can be difficult to know, for some time, whether or not aggressive policies need to be, or retrospectively should have been, followed. For both paths shown, over an extended stretch of time, y^e is low but improving and π^e is below target and falling, with interest rates (not shown) near the ZLB, as the unstable middle steady state (π_L, y_L) is approached. Only then, after a possibly extended period near (π_L, y_L) , does it become evident whether the economy will recover or will instead deteriorate and move deep into the stagnation region. Third, for essentially the same reason, from an initial position (π^e, y^e) outside the domain of attraction, two nearly identical fiscal and monetary policies can have very different outcomes over time even though their impacts are nearly identical for an initial stretch of time.

Finally, as we will see, an aggressive policy change will be required if expectations are quite pessimistic. On the other hand, it is also possible for the policy to be too aggressive, leading to output and inflation dynamics that push expectations to the northeast so far that they are outside the domain of attraction.

3.3 Extension to the stochastic economy

We now examine dynamics under adaptive learning in the model incorporating exogenous stochastic productivity shocks A_t and mark-up shocks ν_t . The shocks are assumed to be independent of each other and to take the form

$$\begin{aligned}\ln(A_t/\bar{A}) &= \rho_A \ln(A_{t-1}/\bar{A}) + \ln(\varepsilon_{A,t+1}) \\ \ln(\nu_t/\bar{\nu}) &= \rho_\nu \ln(\nu_{t-1}/\bar{\nu}) + \ln(\varepsilon_{\nu,t+1}),\end{aligned}$$

where $0 \leq \rho_A, \rho_\nu < 1$, and where $\ln(\varepsilon_{A,t+1}) \stackrel{iid}{\sim} N(0, \sigma_A^2)$ and $\ln(\varepsilon_{\nu,t+1}) \stackrel{iid}{\sim} N(0, \sigma_\nu^2)$. For convenience we assume that the AR(1) parameters $\bar{A}, \bar{\nu}, \rho_A, \rho_\nu$ are known to agents.²⁴

In this stochastic model we modify the forecasting rules to include a dependence on the observable exogenous shocks. Specifically, we assume that agents have a perceived law of motion (PLM) taking the form

$$\begin{aligned}\ln(y_t) &= f_y + d_{yA} \ln(\tilde{A}_t) + d_{y\nu} \ln(\tilde{\nu}_t) + \eta_{yt} \\ \ln(\pi_t) &= f_\pi + d_{\pi A} \ln(\tilde{A}_t) + d_{\pi\nu} \ln(\tilde{\nu}_t) + \eta_{\pi t},\end{aligned}$$

where $\eta_{yt}, \eta_{\pi t}$ are perceived white noise shocks, where $\tilde{A}_t \equiv A_t/\bar{A}$ and $\tilde{\nu}_t = \nu_t/\bar{\nu}$. To form forecasts y_{t+s}^e and π_{t+s}^e at time t agents need to estimate the parameters of the PLM using data up to period $t-1$ and iterate the estimated PLM forward to period $t+s$.

Under constant gain recursive least squares (RLS) learning the coefficient vectors, ϕ_y, ϕ_π where $\phi'_y = (f_y, d_{yA}, d_{y\nu})$ and $\phi'_\pi = (f_\pi, d_{\pi A}, d_{\pi\nu})$, are time-varying and updated over time using recursive least squares regressions of $(\ln(y_t), \ln(\pi_t))$ on $x'_t = (1, \tilde{A}_t, \tilde{\nu}_t)$. The recursive updating equations, which are standard,²⁵ are given by

$$\begin{aligned}\phi_{yt} &= \phi_{y,t-1} + \omega_t \mathcal{R}_t^{-1} x_{t-1} (y_{t-1} - \phi'_{y,t-1} x_{t-1}) \\ \phi_{\pi t} &= \phi_{\pi,t-1} + \omega_t \mathcal{R}_t^{-1} x_{t-1} (\pi_{t-1} - \phi'_{\pi,t-1} x_{t-1}) \\ \mathcal{R}_t &= \mathcal{R}_{t-1} + \omega_t (x_{t-1} x'_{t-1} - \mathcal{R}_{t-1}).\end{aligned}\tag{25}$$

Note that $\phi_{yt}, \phi_{\pi t}$ are updated based on their most recent forecast errors. Here \mathcal{R}_t is an estimate of the second-moment matrix of regressors. RLS updating equations

²⁴This simplifies the agent's problem. If the parameters are unknown it would be straightforward for agents to use consistent estimates of them.

²⁵See, for example, Evans and Honkapohja (2001), Chapter 1, or Evans and Honkapohja (2009).

allow for a time-varying gain ω_t . We will focus on the constant gain case $\omega_t = \omega$ for $0 < \omega < 1$, but will also briefly discuss decreasing gain cases in which $0 < \omega_t < 1$ and $\omega_t \rightarrow 0$.

Letting $f_y, d_{yA}, d_{y\nu}, f_\pi, d_{\pi A}, d_{\pi\nu}$ now denote the time t values of their estimates, expectations of output and inflation s steps ahead, based on the observed exogenous shocks \tilde{A}_t and $\tilde{\nu}_t$, are given by

$$y_{t+s}^e = e^{f_y} \tilde{A}_t^{\rho_A^s d_{yA}} \tilde{\nu}_t^{\rho_\nu^s d_{y\nu}} \text{ and } \pi_{t+s}^e = e^{f_\pi} \tilde{A}_t^{\rho_\pi^s d_{\pi A}} \tilde{\nu}_t^{\rho_\nu^s d_{\pi\nu}},$$

With these expectations, the temporary equilibrium at time t is given by (12), (13), (14), (16) and (17), with $\Xi_t^e = \Xi_t = 1$, subject to the modification described in the following paragraph. The dynamic path under adaptive learning is then specified recursively. At the beginning of time $t+1$ estimates of $(f_y, d_{yA}, d_{y\nu}, f_\pi, d_{\pi A}, d_{\pi\nu})$ are updated to include the time t data point using the RLS equations. Then, after the time $t+1$ exogenous random variables are drawn, the temporary equilibrium equations determine $y_{t+1}, c_{t+1}, \pi_{t+1}$ and R_{t+1} . Given initial conditions and continuing in this way generates an equilibrium time path $\{y_t, c_t, \pi_t, R_t\}_{t=0}^\infty$ for the economy under adaptive learning. For further details on implementation of learning and dynamics see Appendix C.

For our numerical simulations we conduct stochastic simulations over long periods of time and we must allow for trajectories that can go very far from steady states, and consequently we make two modifications in our simulations. First, we assume that after T periods the transitory stochastic component of output and inflation forecasts can be ignored by agents, i.e. we set $y_{t+s}^e = e^{f_y t}$ and $\pi_{t+s}^e = e_t^{f_\pi}$ for $s \geq T$. This is a convenient way of speeding up computations. In our numerical simulations we set $T = 28$. Secondly, we assume agents believe that after T_1 periods the real interest reverts to its steady-state value β^{-1} , i.e. $r_{t+s}^e = \beta^{-1}$ for $s \geq T_1$. Some assumption like this is needed for examining global dynamics since there are some regions of the expectational parameter space for which the expected real interest rate factor would be less than one, implying undefined consumption. In our benchmark simulations we set $T_1 = 20$, i.e. at each time t agents believe real interest rates will return to their steady-state value after five years. While an assumption like this is needed for technical reasons, it should also be viewed as making a substantive assumption about expectations: agents believe that periods of persistently high or low real interest rates will end after five years.²⁶

²⁶Of course, monetary policy can in principle commit to a path of future *nominal* interest rates over a much longer period. In Section 5 we explore the impact of credible forward guidance by the Central Bank about future nominal rates.

Before turning to the numerical results we discuss the role of the gain sequence ω_t . Consider first the decreasing gain case in which $\omega_t \rightarrow 0$, as $t \rightarrow \infty$, at an appropriate rate such as t^{-1} . As with the nonstochastic case, if the variances of the stochastic shocks are not too large, and with some additional plausible assumptions, we can expect there to be fixed forecast parameters $\bar{\phi}_y, \bar{\phi}_\pi$ that correspond to an equilibrium near the targeted steady state. The resulting equilibrium, usually called a “restricted perceptions equilibrium” (RPE), is a generalization of REE: the forecast coefficients $\bar{\phi}$ are minimum mean squared error within the restricted class of linear forecast models used by agents, though in principle better nonlinear forecast rules may exist.²⁷ The RPE also differs from the REE due to our boundedly optimal agents’ use of point-expectations in their forecasting. However the RPE can be viewed as an approximation to the REE centered at the targeted steady state.²⁸

The *E-stability principle* states that, in the decreasing gain case, with suitable additional assumptions, this RPE will be locally stable under RLS learning, so that for initial expectations near the RPE parameters $\bar{\phi}_y, \bar{\phi}_\pi$ we will have $\phi_{yt} \rightarrow \bar{\phi}_y$ and $\phi_{\pi t} \rightarrow \bar{\phi}_\pi$. Similarly, we can expect an RPE at the stagnation steady state to be locally stable but for the middle steady state (π_L, y_L) to be locally unstable under RLS learning. Thus in the stochastic model, we expect local stability of the equilibrium paths under RLS learning to be inherited from E-stability of the steady states.

In practice, in applied macro models, a constant gain $\omega_t = \omega$ with $0 < \omega < 1$, is almost invariably assumed. This allows agents to track structural change and changes in policy, but also results in “perpetual learning dynamics” around an REE or RPE. An advantage of this in empirical applications is that the learning dynamics are part of a stationary system.²⁹ In our numerical simulations we employ constant gain learning. Some general theoretical results, known as stochastic approximation results, are available for constant-gain learning in the stochastic model for the limiting case of $\omega > 0$ sufficiently small³⁰, based on an ordinary differential equation

²⁷For examples and discussion of RPE see, e.g. Evans and Honkapohja (2001), Ch. 13, and Branch (2006). Applications of RPE in nonlinear models are given in Evans and McGough (2020a) and Evans and McGough (2020b).

²⁸For shocks A_t, ν_t with finite support, the REE and RPE coincide in the limit $\sigma_A, \sigma_\nu \rightarrow 0$ and $\rho_A, \rho_\nu \rightarrow 0$.

²⁹Discussion and applications of constant-gain learning in economics include Sargent (1999), Evans and Honkapohja (1993), Cho, Williams, and Sargent (2002), McGough (2006), Milani (2007), Orphanides and Williams (2007), Branch and Evans (2011) and Eusepi and Preston (2011).

³⁰See e.g. Evans and Honkapohja (2001), Section 7.4 of Ch. 7 and Ch. 14, and Evans and Honkapohja (2009). For the related topic of escape dynamics see Cho, Williams, and Sargent (2002) and Williams (2019).

(ODE) approximation to (25).

In the current setting the E-stability principle confirms local stability of both the targeted and the stagnation steady state, and local instability of the middle steady state.³¹ However, under constant gain learning, e.g. for the targeted steady state, there will not be full convergence to the associated RPE, because the forecast rule parameters $\phi_{yt}, \phi_{\pi t}$ will remain stochastic over time, with means near their RPE values and variances approximately proportional to the gain ω . Stochastic approximation results based on the ODE approximation to (25) can be used to compute the “mean dynamics” globally, of the system (25), for initial parameters $\phi_y^0, \phi_\pi^0, \mathcal{R}^0$. However, in practice it is convenient to study the dynamics directly using stochastic simulations.

We are particularly interested in how the size of an initial pessimistic expectations shock affects whether the economy returns to the targeted steady state RPE or whether it is pushed into the stagnation regime along a path toward the stagnation steady state. To study this using simulations of our calibrated model, we consider the impact over time of an unmodelled adverse shock to output expectations y_0^e , such as might have occurred following the 2007-8 financial crisis, lowering agents’ estimates of future output and incomes.

We focus here on the nature of output and inflation dynamics resulting from an expectations shock to y^e under unchanged policy. Assume the economy is initially in the targeted steady state (with $y^e = y^* = 1.00003 \approx 1$) when a shock to expectations occurs and initiates a path of temporary equilibria through learning and the adjustment of expectations. Because our model is now stochastic, we anticipate that, at least for a range of initial y_0^e , whether the economy returns to the targeted steady state will itself be stochastic.

Under adaptive learning dynamics the gain parameter must be specified and in our numerical simulations we set this to $\omega = 0.01$. For long-horizon models, because of the high sensitivity of temporary equilibrium output and inflation to long-run expectations, the gain is typically set somewhat lower.³² However, in the presence of a large shock to the economy, and with a possible change in policy, a higher gain is warranted to track the evolving data.

Consider first a small negative shock to y^e which is inside the domain of attraction in Figure 4. A shock of 0.2 percent to steady-state output expectations (or its present value equivalent), with $\pi^e = \pi^*$ unchanged, shifts output expectations to $y_0^e = \lambda y^*$, where $\lambda = 0.998$. In this case the economy will eventually converge back with very

³¹The intercepts of the expectations functions govern the evolving means of y_t and π_t in the sequence of temporary equilibria, so that the preceding E-stability analysis remains central to the model’s dynamics.

³²See, e.g. Eusepi and Preston (2011).

high probability to the targeted steady state. This is as expected since this shock places expectations substantially inside the domain of attraction: as previously noted the lower boundary of the domain of attraction is approximately $y^e = 0.98792$ when $\pi^e = \pi^*$.

Larger adverse shocks to y_0^e lead to an increasing likelihood of failing to return to the targeted steady state under unchanged policy. The key results are shown in Table 1. For $\lambda = 0.9975$ the probability of convergence to the targeted steady state is 69% percent and for $\lambda = 0.99745$ this probability is only 15%. Thus for a range of expectation shocks the dynamics of the economy can depend sensitively on the sequence of exogenous random shocks affecting output and inflation.

The numerical results seen in Table 1 show that failure to converge to the targeted steady state arises even for pessimistic output expectations well inside the theoretical E-stability domain of attraction shown in Figure 4. The discrepancy for initial expectations inside the nonstochastic domain of attraction in Figure 4 arises for several reasons. First, our assumption $T_1 = 20$ has a sizable effect. Additional simulations show that at $\pi^e = \pi^*$ the lower boundary y^e to the numerical domain of attraction of the targeted steady state falls as T_1 increases. The intuition for this is as follows. With $\pi^e = \pi^*$ and $y^e < y^*$, expected nominal and real interest rates are lower, raising demand and output. This stabilizing effect of monetary policy, in the face of pessimistic output expectations, is blunted, however, because we impose that expected real interest rates are expected to return to the steady-state value after a finite number of periods T_1 . Central banks are not able, of course, to directly control real interest rates, and we view our finite T_1 assumption as plausible. Qualitative results are not affected, of course, by the precise choice of T_1 .

There are two other factors that arise from our stochastic set-up. In the nonstochastic model generating Figure 4 there are only two parameters, corresponding to the intercepts of the RLS system (25). In our stochastic set-up there are six parameters in ϕ_y, ϕ_π , as well as additional parameters in the estimated second-moment matrix \mathcal{R} . The global ODE approximation to (25) thus differs from the E-stability dynamics shown in Figure 4 because of its higher dimensionality. In addition, with constant gain learning the mean dynamics corresponding to the ODE are only be a good approximation for $\omega > 0$ very close to zero and can differ significantly for values even as small as $\omega = 0.01$. In consequence the combination of constant gains $\omega = 0.01$ and stochastic intrinsic shocks leads to sufficient variation in (y_t, π_t) over time so that expectations are more frequently pushed into unstable trajectories.

The key numerical findings are clearly consistent with our general theoretical results. The targeted steady state is locally stable under LS learning, but it is not globally stable. For sufficiently pessimistic initial output expectation shocks,

i.e. $0 < \lambda < 1$ sufficiently low, the proportion of trajectories that converge to the targeted steady state is near zero. In our stochastic set-up this arises for $\lambda \leq 0.9974$. The numerical results are shown in Table 1.

Initial expectation	$P(\text{target})$	$P(\text{stagn.})$
$y_0^e/y^* = 0.9980$	100	0
$y_0^e/y^* = 0.9975$	69	31
$y_0^e/y^* = 0.99745$	15	85
$y_0^e/y^* = 0.99742$	1	99
$y_0^e/y^* = 0.9974$	0	100

Table 1: Percentage convergence to target or stagnation under unchanged policy
100 replications, gain = 0.01

These results illustrate, first, that with constant fiscal policy in the stochastic model there are situations where the long-run outcome may be either the targeted steady state or stagnation depending on the realization of the exogenous random shocks A_t and ν_t . Second, on a formal level we see that the global E-stability analysis of Section 3.2, based on nonstochastic one-parameter PLMs, provides key, though approximate, results concerning convergence of real-time constant-gain RLS learning in the stochastic model with PLMs that depend on exogenous observables.

To understand the magnitude of the expectation shocks given in Table 1, it is helpful to consider a reinterpretation of the role of y^e in the temporary equilibrium model. For the consumption function in Section 2.3, assuming the representative agent case with $\Xi_t^e \equiv 1$, it can be seen that consumption, and hence temporary equilibrium output y_t , depend to first-order on $\{y_{t+s}^e\}_{s=1}^\infty$ through its present value

$$\text{PV}(\{y_{t+s}^e\}_{s=1}^\infty) = \sum_{s=1}^{\infty} (D_{t,t+s}^e)^{-1} y_{t+s}^e \approx \sum_{s=1}^{\infty} \beta^s y_{t+s}^e.$$

We have interpreted steady state learning as agents acting as if $y_{t+s}^e = y_t^e$ for all horizons $s = 1, 2, 3, \dots$. However this is behaviorally equivalent to assuming that agents have an expected output profile with the same present value as $\text{PV}(\{y_{t+s}^e = y_t^e\}_{s=1}^\infty)$.

In particular suppose that agents believe $y_{t+s}^e = \hat{y} < y^*$ for $s = 1, \dots, L$ periods, interpreted as quarters, followed by $y_{t+s}^e = y^*$ for $s > L$. This would correspond to a recession of L periods followed by a return to targeted steady state output. Let $\% \Delta y^e = (y^* - y^e)/y^*$ and $\% \Delta \hat{y} = (y^* - \hat{y})/y^*$. Then the PV of the L -period recession

output expectation sequence equals the PV of a constant sequence $y^e < y^*$ when

$$\% \Delta \hat{y} = (1 - \beta^L)^{-1} \times \% \Delta y^e.$$

Setting $L = 8$, i.e. eight quarters, the numerical lower y^e boundary of the domain of attraction (at $\pi^e = \pi^*$) for the targeted steady state corresponds to an expected recession approximately equal to a 3.4% reduction of expected GDP for two years, followed by a return to normal trend. This assumes unchanged fiscal policy, i.e. constant government spending and taxes, and unchanged monetary policy given by the specified Taylor-rule. In the next section we show that substantially larger pessimistic expectation shocks can be overcome by active fiscal policy.

In interpreting these results it is important to bear in mind that we are employing a benchmark New Keynesian model without capital and without additional frictions often employed in serious empirical DSGE models, such as indexation, habit persistence and adjustment costs. It is possible that extensions like these, which introduce inertia into the dynamics, would enlarge the domain of attraction, without, however, altering the qualitative features of our model in which there are three steady states, including a locally stable targeted steady state and a stagnation region.

4 Fiscal Policy

We turn now to fiscal policy. A growing literature has been reconsidering the effects of fiscal policy in light of the relatively large fiscal stimuli adopted in various countries in the aftermath of the Great Recession. For example, Christiano, Eichenbaum, and Rebelo (2011), Corsetti, Kuester, Meier, and Muller (2010) and Woodford (2011) demonstrate the effectiveness of fiscal policy in models with monetary policy when the ZLB on the interest rate is reached. For a contrary view see Mertens and Ravn (2014). Most of this literature makes the RE assumption. The AL literature has shown that quite different results can arise both in NK and Real Business Cycle models; see Evans, Guse, and Honkapohja (2008), Benhabib, Evans, and Honkapohja (2014), Mitra, Evans, and Honkapohja (2013), Gasteiger and Zhang (2014) and Mitra, Evans, and Honkapohja (2019).

We examine fiscal policy under AL. It is assumed that initially the economy is at the targeted steady state when a pessimistic shock to expectations impacts the economy, which then begins to adjust through learning. For concreteness this shock is modelled as a negative shock to output expectations y^e (other cases of shocks could be studied). We direct our attention to negative expectation shocks sufficiently large so that without policy change the path of the economy would with high probability fail

to return to the targeted steady state and would instead be trapped in the stagnation region.³³ We consider two “large shock” cases. The first case is $y^e = 0.997$ which, based on Table 1, is big enough to result in convergence to the stagnation steady state almost 100% of the time in our calibrated stochastic model.³⁴ In the second case the negative expectation shock is much larger, $y^e = 0.991$. Using the present value interpretation of the expectation shock given at the end of the preceding section, the shock $y^e = 0.997$ corresponds to an expected two-year recession of 3.9% of GDP, while the larger shock $y^e = 0.991$ corresponds to an expected two-year recession of 11.7% of GDP.

Because we assume Ricardian households, we examine the impact of changes in the level of government purchases and focus on temporary increases in the level of government spending on goods and services.³⁵ When there is a change in fiscal policy, agents take account of the tax effects of the announced path of policy. Given the Ricardian assumption, we can assume balanced budget increases in spending so that the path of taxes matches the path of government spending.

Evidently, fiscal policy needs to be tuned to the size of the exogenous expectations shock. We first consider the evolution of an economy following the negative output expectations shock $y_0^e = 0.997 \times y^*$, with initial inflation expectations unchanged at the target level $\pi_0^e = \pi^*$. Then at $t = 1$ the government announces an increase in government spending for T_p periods, i.e.

$$g_t = \Upsilon_t = \begin{cases} \bar{g}', t = 1, \dots, T_p \\ \bar{g}, t \geq T_p + 1, \end{cases}$$

where $\bar{g}' > \bar{g}$. Thus government spending and taxes are changed in period $t = 1$ and this change is reversed at a later period $T_p + 1$. We assume that the announcement is fully credible and the policy is implemented as announced. These assumptions could, of course, be relaxed at the cost of added complexity in the analysis.

Using stochastic simulations we study the paths of the economy, under AL, after a pessimistic shock and examine the potential role for fiscal policy to prevent stag-

³³ Analogous simulations could of course be done when the y_0^e shock is smaller and there is eventual convergence to targeted steady state without change in policy. The question of interest would then be whether fiscal policy can speed up the recovery back to the targeted steady state.

³⁴ As noted above, we use the gain $\omega = 0.01$ for our simulations. The size of the appropriate gain reflects a trade-off between tracking and filtering. Our choice reflects a balance between a larger gain, needed to track the impact of unusual shocks and/or changes in policy, and a smaller gain needed to filter random shocks and stabilize the economy.

³⁵ In further work it would be of interest to introduce alternative fiscal frameworks with distortionary taxes and/or public debt.

nation or ameliorate bad outcomes.³⁶ Our main focus is whether fiscal policy can alter the dynamic path so that there is instead convergence to the targeted steady state. We will see that the impact of fiscal policy may depend critically on the size and length of fiscal policy. In addition, the sequence of random shocks A_t and ν_t have an impact on the success of fiscal policy.

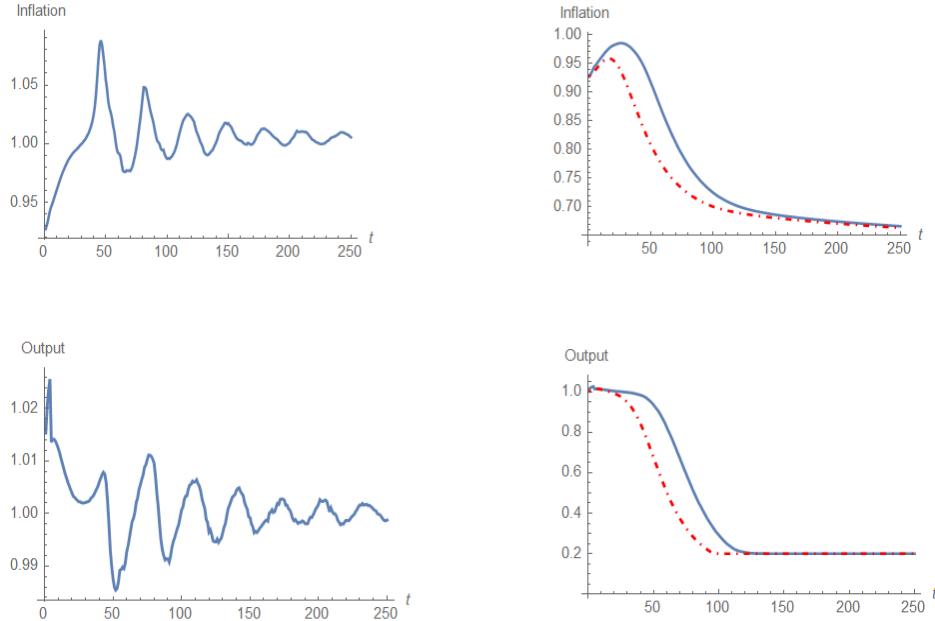


Figure 5: Figures on the left show mean inflation (output) paths when policy is successful. Right hand figures show paths of mean inflation (output) when policy is unsuccessful (solid curve) and paths without policy (dashed curve).

As an illustration, means for simulations of time paths are illustrated in Figure 5 for a fiscal stimulus with $T_p = 4$, i.e. one-year policy, and $\bar{g}' = 0.221875$, an 11.1% temporary increase in government spending. This choice of (\bar{g}', T_p) is made because, depending on the sequence of random shocks over the simulation period, the policy may or may not be successful in avoiding stagnation. The other parameters are standard, see in section 3.2. The dynamics are replicated for 100 draws of the random shocks.

In this example, the probability of convergence to target steady state is about 59 percent. In Figure 5, the left hand figures show the mean paths of inflation

³⁶We emphasize that these simulation results are designed to be illustrative, i.e. to exhibit the range of possible results that can be obtained in our model. Using the model to fit actual historical episodes is reserved for future research.

and output, when the mean is over the replications that result in convergence to the targeted steady state. The right hand figures show the corresponding dynamics when the mean is computed over the replications that result in divergence from the target (and eventual convergence to the stagnation steady state). For comparison, the latter figures also contain the mean path when there is no policy change. This example illustrates that whether or not the fiscal policy is successful in pushing the economy back to the targeted steady state can sometimes depend sensitively on the sequence of stochastic shocks hitting the economy over time.

We remark that in the successful cases, in which with the fiscal stimulus aggregate output and inflation converge to the targeted steady state, mean paths show significant cyclical fluctuations in GDP and the quarterly inflation rate. This is a reflection of the cyclical dynamics under adaptive learning under standard policy within the domain of attraction of the targeted steady state. It is possible that more complex, and possibly state-dependent, fiscal and monetary policies would lead to smoother convergence, and optimal policy within the present set-up would be a natural topic for future research. However we also emphasize, as earlier noted, that our analysis has been conducted within the simple and standard canonical New Keynesian model, which is purely forward-looking. Adding additional state dynamics and frictions of the type often used in empirical computational models is likely to smooth paths.

A systematic analysis of the case $y_0^e = 0.997 \times y^*$ is now conducted by varying the magnitude and length of fiscal policy and studying the estimated probability of the economy going back to target steady state. In general, temporary increases in g are effective in raising output. Small temporary increases in g lead only to temporary increases in y , but larger temporary increases in g can shift the economy back to a path converging to the targeted steady and resulting in a permanent increase in output relative to the paths that would be followed without the fiscal stimulus. Table 2 shows the results for the expectation shock $y_0^e = 0.997 \times y^*$, $\pi_0^e = \pi^*$ for alternative values of both the length T_p and the magnitude \bar{g}' of fiscal policy. Table 2 gives the estimated probabilities of convergence to the targeted steady state (vs eventual convergence to the stagnation steady state).

$T_p \setminus \bar{g}'$	0.2	0.225	0.25	0.275	0.3	0.325	0.35	0.375	0.4
1	0	0	0	0	94	100	100	100	100
2	0	0	97	100	100	100	100	100	100
3	0	0	100	100	100	100	100	100	100
4	0	95	100	100	100	100	100	67	1
5	0	99	100	100	100	100	5	0	0
6	0	100	100	100	100	2	0	0	0
7	0	100	100	100	4	0	0	0	0
8	0	100	100	44	0	0	0	0	0
9	0	100	100	3	0	0	0	0	0
10	0	100	96	1	0	0	0	0	0
11	0	100	30	1	0	0	0	0	0
12	0	100	5	0	0	0	0	0	0

Table 2: Percentage of simulations in which fiscal policy successfully results in convergence to the targeted steady state starting from pessimistic output expectations $y_0^e = 0.997 \times y^*$. Based on 100 replications with length 500.

Table 2 shows that the sequence of serially correlated random productivity and mark-up shocks can matter: for a fiscal policy that is usually successful, a particularly unfavorable sequence of shocks can adversely affect expectations enough to prevent the policy from working. However, for a substantial range of policies, in particular for \bar{g}' between 0.25 and 0.35 with T_p between 2 and 4 quarters, a fiscal stimulus is successful nearly 100% of the time. In these cases the cumulative fiscal spending multipliers would of course be huge, reflecting the fact that a temporary fiscal stimulus prevents the economy from descending into stagnation and pushes it back toward convergence to the targeted steady state.

It can also be seen that in many cases a fiscal stimulus that is too long can be counterproductive. For example, for $g = 0.30$ the effectiveness of the stimulus decreases greatly if T_p is increased to $T_p = 7$ quarters or longer. This is a reflection of the negative effect on consumption of the tax burden associated with higher government spending, which we assume is correctly foreseen by households. In particular, the impact on aggregate output is largest in the first period when a fiscal policy of a given magnitude Δg for T_p periods is initiated. In this case the present value of the tax burden is simply Δg and the direct impact of this on consumption is $-(1 - (1 - \xi)\beta)\Delta g$, which is small compared to the increase in aggregate demand for output from government spending Δg . For larger T_p the present value of the tax burden is larger; consequently the reduction in consumption in the initial period

is greater, leading to a smaller initial increase in aggregate output and inflation. Against this, of course, a larger T_p means that the increase in demand continues for a longer period of time, so that under learning expectations will adjust to a greater extent to the higher values of output and inflation realized during the policy.

We now examine case of the larger expectations shock $y_0^e = 0.991 \times y^*$. Following this shock a temporary fiscal stimulus is applied with government spending increased from $\bar{g} = 0.2$ to $\bar{g}' = 0.3, \dots, 0.7$ for $T_p = 1, \dots, 6$ quarters. Table 3 shows the probability (in percentages) of cases where the policy is successful. The success probabilities are generally lower than those in Table 2, and values of \bar{g}' need to be significantly larger than those in Table 2 in order to be successful. However, there are still policies with a high degree of success: the highest success rate shown in Table 2 is 94%.

$T_p \setminus \bar{g}'$	0.3	0.35	0.4	0.45	0.5	0.55	0.575	0.6	0.625	0.65	0.675	0.7
1	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	39	75	80	91
4	0	0	0	0	0	83	89	94	90	83	40	11
5	0	0	0	9	92	65	22	3	1	0	0	0
6	0	0	0	86	29	1	0	0	0	0	0	0

Table 3: Percentage of simulations in which fiscal policy successfully results in convergence to the targeted steady state starting from very pessimistic output expectations $y_0^e = 0.991 \times y^*$. Based on 100 replications in each cell.

We find from these results that a sufficiently large stimulus of appropriate duration can have a high probability of extracting the economy back to convergence to target even if pessimistic output expectations are fairly deep inside the stagnation region. However, a higher probability of avoiding the stagnation regime can be achieved, with a much smaller stimulus, if the policy is implemented when expectations are less pessimistic. This suggests that following a large adverse shock to expectations, in which there is major risk of the economy descending into the stagnation regime, a fiscal stimulus should be implemented as early as possible.³⁷ This is discussed below in Section 5.2.

A general implication of our fiscal policy results, which is evident but worth emphasizing, is that the size and impulse response profile of the government spending

³⁷This accords with testimony by Lawrence Summers to the Joint Economic Committee hearing on 16 January 2008, that fiscal “stimulus program should be timely, targeted and temporary.”

multiplier depends sensitively on both the current state of expectations, when the policy is initiated, and nonlinearly on the size and duration of the spending increase.

5 Extensions

We now briefly consider in turn several extensions relevant to designing policies to avoid stagnation.

5.1 Including forward guidance in monetary policy

In the preceding Section it was seen that fiscal policy is not always successful in the sense of guaranteeing that the economy escapes the stagnation regime and converges to the target steady state. Moreover, the probability of success becomes lower if the shock to the economy is more pessimistic, and more aggressive fiscal policy may then be needed. It therefore makes sense to ask whether expansionary unconventional monetary policy can help.

The current framework is well suited to analyze forward guidance, which of course has been a form of unconventional monetary policy that central banks used during and following the Great Recession.³⁸ We model forward guidance as a commitment by the central bank to keep the policy interest rate at the ZLB for the first T_m periods after the expectation shock occurs. With forward guidance the interest rate rule (16) becomes

$$R_t = \begin{cases} 1, & t = 1, \dots, T_m \\ R(\pi_{t+1}^e, y_{t+1}^e), & t \geq T_m + 1. \end{cases}$$

Focusing on very pessimistic initial conditions we examine whether combining forward guidance with fiscal policy can increase the probability of getting back to the targeted steady state when compared to using only fiscal policy or only forward guidance.

To study this we consider initial output expectation shocks even more pessimistic than used in Table 3. We first set $y_0^e = 0.985 \times y^*$, which corresponds to an expected two-year recession of 19.5% of GDP. Without a change in policy, the economy always, in our simulations, converges toward the stagnation steady state. To look at recovering from this expectation shock, we explore various settings of temporary fiscal stimulus \bar{g}_1 , T_p combined with forward guidance T_m . We continue to use 100 stochastic simulations for each policy setting.

³⁸Without extensions the model is not suited to analyzing other forms of unconventional policies, such as large scale asset purchases.

If only forward-guidance monetary policy is used, without including a fiscal stimulus, the probability of convergence to the targeted steady state is zero for $T_m \leq 10$ or $T_m \geq 15$. The probability of success is positive for $T_m = 11$ (43%), $T_m = 12$ (25%) and $T_m = 13$ (11%). If instead only fiscal stimulus is employed, the probability of convergence to the targeted steady state is close to zero except, again, for a specific cases for fiscal policy settings: $\bar{g}_1 = 0.55$ with $T_p = 8$ (38%), $\bar{g}_1 = 0.6$ with $T_p = 7$ (43%), $\bar{g}_1 = 0.65$ with $T_p = 6$ (40%), $\bar{g}_1 = 0.7$ with $T_p = 6$ (35%), $\bar{g}_1 = 0.75$ with $T_p = 5$ (60%), $\bar{g}_1 = 0.75$ with $T_p = 5$ (60%) and $\bar{g}_1 = 0.8$ with $T_p = 5$ (53%).³⁹ Thus success can be achieved by using only fiscal policy, but the probability of failure is over 50% except for some quite large \bar{g}_1 .

Better outcomes can be achieved by combining fiscal policy and forward guidance. Table 4 illustrates the results from detailed analysis of the case with $y_0^e = 0.985 \times y^*$ in which forward guidance setting $T_m = 6$ is combined with different fiscal stimulus settings. The highest probability of success (convergence to the targeted steady state) is 73% with $\bar{g}_1 = 0.55$ with $T_p = 3$. Similar results are obtained for nearby values of T_m (further results are in Appendix G.1). Thus for the case of severely depressed output expectations, $y_0^e = 0.985 \times y^*$ there is a significant increase in the probability of escape from stagnation when both policies are actively employed.

$T_p \setminus \bar{g}_1$	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8
1	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	37	64	60	69	67	41
3	0	0	0	45	62	73	36	12	3	1	0
4	0	0	60	69	37	6	0	1	0	0	0
5	0	32	65	30	4	0	0	0	0	0	0
6	0	64	48	4	0	0	0	0	0	0	0
7	0	57	7	1	0	0	0	0	0	0	0
8	31	36	0	0	0	0	0	0	0	0	0

Table 4: Percentage of convergence to target for different specifications of combined policy based on 100 replications for $y_0^e = 0.985 \times y^*$, $T_m = 6$.

For even more pessimistic expectation shocks, it can happen that either fiscal policy or forward guidance alone are ineffective in moving the economy to the targeted steady state, while use of combined policy can still achieve some success. As a final example, consider initial output expectations $y_0^e = 0.98 \times y^*$. This corresponds to an

³⁹Cases where probability of success is less than 10 percent are not reported.

expected two-year recession of 25.9% of GDP. In this case forward guidance alone is totally ineffective (for $T_m = 1, \dots, 20$). Using fiscal policy alone is also largely ineffective: in the range $\bar{g}_1 = 0.25, \dots, 0.90$ there are only a few cases with positive probability for convergence to target, and the highest probability is 28% for $\bar{g}_1 = 0.85$ with $T_p = 6$. However, combined policy improves the chances of converging to the targeted steady state. In Table 5 the highest probability of convergence is 45% when $\bar{g}_1 = 0.6$, $T_p = 5$ and $T_m = 7$.⁴⁰ (More results are given in a set of Tables in the Appendix G.2 and in Appendix G.2.1 for very high values of \bar{g}_1 .)

$T_p \setminus \bar{g}_1$	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8
3	0	0	0	0	0	0	0	17
4	0	0	0	0	21	40	29	14
5	0	0	0	45	25	6	0	0
6	0	0	41	15	0	0	0	0
7	0	37	11	1	0	0	0	0
8	22	11	1	1	0	0	0	0

Table 5: Percentage of convergence to target for different specifications of combined policy for $y_0^e = 0.98$, $T_m = 7$ and 100 replications.

The results of this section show that, for very pessimistic output expectations, adding forward guidance to fiscal policy can substantially improve the chances of converging to the targeted steady state, at least for the wide range of fiscal policies we considered. A different approach might be to use an even larger fiscal stimulus for which there is some improvement but the results are not very encouraging and require very large increases in \bar{g} .⁴¹

5.2 Delays in policy

In Section 4 we suggested that in the face of a large pessimistic expectations shock it may be important to implement a fiscal stimulus quickly. We here briefly illustrate the effect of policy delays. For a given output expectations shock y_0^e we consider the

⁴⁰The table has been trimmed from the corresponding table in the Appendix by eliminating rows and columns with only zeroes.

⁴¹For fiscal policy alone and very pessimistic expectations $y_0^e = 0.98 \times y^*$ the highest probabilities we obtained were 30% for $\bar{g}_1 = 1.05$ with $T_p = 5$ and $\bar{g}_1 = 1.25$ with $T_p = 4$; in the case $\bar{g}_1 = 1.30$ with $T_p = 4$ the probability is 33%. These are levels of government spending greatly in excess of the targeted steady state level.

effect on the probability of success of a delay by T_s periods. We restrict attention to the case, examined earlier in Table 3, in which initial output expectations are very pessimistic, given by $y_0^e = 0.991 \times y^*$, and in which we now assume that policy is executed with a delay of 4 periods (one year). Table 6 reports the relevant part of the table, i.e. ranges $T_p = 3, \dots, 6$ and $\bar{g}' = 0.45, \dots, 0.7$ based on 100 replications.⁴²

$T_p \setminus \bar{g}'$	0.45	0.5	0.55	0.575	0.6	0.625	0.65	0.675	0.7
3	0	0	0	0	0	0	0	0	0
4	0	0	0	0	4	48	53	24	15
5	0	0	51	32	12	6	0	1	0
6	0	39	7	1	1	0	0	0	0

Table 6: Percentage of simulations in which fiscal policy \bar{g}' for T_p periods and delay of $T_s = 4$ periods results in convergence to target from pessimistic output expectations $y_0^e = 0.991 \times y^*$. Based on 100 replications with length 500.

It is seen that the percentage of success with delay is generally lower than the corresponding percentage when there is no delay. Most noticeably, if we compare no-delay fiscal policies, with the highest chance of success, with six-period delayed policies with the highest probability of success, the probability of success falls from 94% to 53%. The reason for this is that during the period of delay output expectations deteriorate further and inflation expectations also begin to decline.

5.3 Credit frictions and calibration of the discount factor

In Section 3.1 we noted that at the low steady state (π_L, y_L) the (gross) policy interest rate is approximately equal to one while the (gross) inflation rate is approximately equal to β . From Figures 3 and 4 it is evident that π_L plays a key role in the expectation dynamics since the unstable steady state (π_L, y_L) is on the edge of the domain of attraction of the targeted steady state and for $\pi^e < \pi_L$ and $y^e < y_L \approx y^*$ the economy lies within the stagnation trap. The appropriate calibration of the discount factor β is thus worth discussion. Our numerical results have used the quarterly calibration of $\beta = 0.99$ which implies a quarterly deflation rate at π_L of 1%.

While $\beta = 0.99$ is fairly standard, there are good reasons to consider alternative, higher, values. The historical average realized net real interest rate on US Treasuries bills is not more than 1% per annum. In an economy without growth this corresponds

⁴²The policy thus starts in period 5 and ends in period $5 + T_p$.

to a discount factor of about $\beta = 0.9975$.⁴³ The critical inflation rate at the edge of the stagnation trap at $y^e < y_L$ is then an annual deflation rate of 1%.

A second factor that can lead to a higher level of the critical inflation rate is the existence of credit frictions. Various models have been proposed that generate a spread between different interest rates on loans. A prominent example within a NK setting is described in Curdia and Woodford (2010) and developed at length in Curdia and Woodford (2015). Their framework posits a heterogeneous agents set-up with two types of household, at any given time, experiencing different realizations of taste shocks. This leads to lending from agents who are currently more patient than those who are currently more impatient. Frictions in the financial intermediation sector result in a borrowing rate above the lending rate.

Embedding a heterogeneous agents framework into our model is beyond the scope of the current paper. However, it is natural to incorporate a shortcut, motivated by Woodford (2011), which is to assume that the market interest rate relevant in household Euler equations for the “intertemporal allocation of expenditure is not the same as the central bank’s policy rate” (Woodford, p. 16). Woodford (2011) and Curdia and Woodford (2015) focus on the implications of the time variation in this spread, while for our purposes the key implication is a positive steady state spread $\varphi = R - i > 0$, where i is the policy rate and R is the interest rate relevant for household decision-making. The benchmark calibration in Curdia and Woodford (2015) corresponds to a value $\varphi = 0.0025$, i.e. to 1% per annum.

With credit frictions we assume a spread $\varphi > 0$ between the market rate R_t and the policy rate $R_t - \varphi$. Since the policy rate obeys the ZLB for net interest rates, the market interest rate factor relevant for the consumption Euler equation satisfies $R_t \geq 1 + \varphi$. The interest-rate rule (16), with inflation target π^* , is then replaced by

$$R_t = 1 + \varphi + (R^* - (1 + \varphi)) \left(\frac{\pi_{t+1}^e}{\pi^*} \right)^{BR^*/(R^* - (1 + \varphi))} \left(\frac{y_{t+1}^e}{y^*} \right)^{\phi_y}. \quad (26)$$

The positive spread φ increases the low steady-state inflation rate to $\pi_L \approx \beta(1 + \varphi)$. This has a number of implications, one of which is particularly relevant for policy: if $\beta(1 + \varphi) > 1$ then it is possible to have $1 < \pi_L < \pi^*$, so that the critical inflation rate at (π_L, y_L) is a zero or low positive inflation rate, rather than a deflation rate.

⁴³We note that Eggertsson (2010) uses a calibration of $\beta = 0.997$ in a model of the US economy during the Great Depression. During the Great Depression deflation reached 10% per year during the trough.

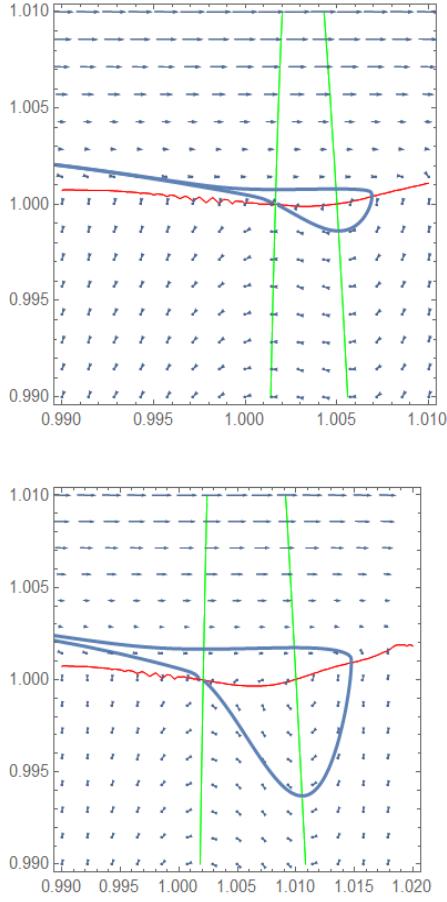


Figure 6: The top panel shows the domain of attraction for the case $\beta = 0.9975$, $\varphi = 0.0025$ and $\pi^* = 1.005$. The bottom panel shows the domain of attraction for the case $\beta = 0.9975$, $\varphi = 0.0025$ and $\pi^* = 1.01$. π^e is on the horizontal and y^e on the vertical axis.

The central consequences of a credit spread can be seen by comparing the domains of attraction of π^* with and without a credit spread. The top panel of Figure 6 illustrates the domain of attraction of the targeted steady state for the model with a high subjective discount rate and a positive credit spread. The domain of attraction is now significantly smaller than that in the basic model.⁴⁴ At $\pi^e = \pi^*$ the value of y^e

⁴⁴The truncation of expected real interest rates to a finite horizon in consumer optimization is employed because a wide state space is needed for the analysis. Here we set T_1 to a fairly high value, $T_1 = 100$, in order to reduce its numerical impact. See Section 3.3 for discussion of T_1 . Using finite T_1 reduces somewhat the domain of attraction.

at the low boundary of the domain of attraction is approximately $y^e = 0.9986$, much higher than the corresponding value in Figure 4. Similarly at $y^e = y^*$, the value of π^e at the low boundary of the domain of attraction is π_L and now corresponds to positive net inflation. Thus the impact of a higher discount factor and a positive credit spread is to reduce the size of the domain of attraction of (y^*, π^*) , making the targeted steady state less stable.

Raising the inflation target π^* can be used to partially offset the effects of the credit spread. The bottom panel of Figure 6 shows the domain of attraction for the higher inflation target $\pi^* = 1.01$ (corresponding to 4% inflation target annually). It can be seen that the domain of attraction in the bottom panel is markedly bigger than the one in the top panel in Figure 6. In particular, at $\pi^e = \pi^* = 1.01$, the value of y^e at the low boundary of the domain of attraction is approximately $y^e = 0.99376$, much lower than the corresponding value shown in the top panel of Figure 6 at $\pi^e = \pi^* = 1.005$. Similarly the width of the domain of attraction at $y^e = y^*$ is much larger in the bottom panel of Figure 6 than in the top panel of the same figure. It is still the case, however, that at $y^e = y^*$, the value of π^e at the low boundary of the domain of attraction corresponds to positive net inflation.

In summary, credit frictions reduce the domain of attraction of the targeted steady state. Our examples have focused on the possibility of a stagnation trap when initial output expectations are low. However low inflation expectations are also a reason for concern. Taking into account credit frictions, expected inflation rates significantly below the central bank target, even if positive, raise the possibility of a path toward stagnation and the possible need for aggressive policy. A higher inflation target appears to be one way to improve the robustness of the targeted steady state under normal policy.⁴⁵ Nonetheless, the qualitative aspects of dynamics shown in Figures 3 and 4 remain unchanged.

5.4 Blended expectations

Inflation targeting has been practiced by a substantial number of Central Banks since it was formally adopted by New Zealand and Canada in 1990 and 1991. In our numerical calibrations we have used a target of 2%, which, for example was formally adopted by the Bank of England in 2003. The target of 2% in the US was formally announced by the Federal Reserve in January 2012, bringing it in line with a number of other countries, but this was preceded by a period in which 2% was believed to be the Fed's informal target. One of the main reasons given for having an explicit

⁴⁵However, Branch and Evans (2017) emphasize that an increase in the inflation target must be done carefully to avoid de-anchoring of inflation expectations under adaptive learning.

inflation target is that this can anchor expectations, so that expected inflation is less sensitive to observed inflation rates or exogenous shocks.

It is certainly possible that having an explicit inflation target helps anchor expectations, e.g. see Gurkaynak, Swanson, and Levin (2010). Against this Branch and Evans (2017) have argued that policymakers should take into account that expectations can become de-anchored by observed economic data. In this section we take a balanced approach to this issue by considering “blended expectations,” in which inflation expectations are a weighted average of the forecasts arising from our adaptive learning rules and the inflation target set by the central bank. Thus we now set

$$\pi_t^e = \varpi \tilde{\pi}_t^e + (1 - \varpi) \pi^*, \text{ for } 0 < \varpi < 1, \quad (27)$$

where ϖ is the weight placed on the adaptive learning forecast $\tilde{\pi}_t^e$ and $1 - \varpi$ is the weight on the central bank inflation target.

We now look at global E-stability dynamics with blended expectations in comparison with the benchmark case given in Section 3.2.⁴⁶ Temporary equilibrium is defined by the earlier equations with given expectations (π_t^e, y_t^e) :

$$\pi_t = G_1(y_t, y_t^e) \text{ and } y_t = G_2(\pi_t^e, y_t^e).$$

As usual, the E-stability differential equations are

$$\frac{dy^e}{d\tau} = F_y(\pi^e, y^e) \equiv G_2(\pi^e, y^e) - y^e$$

and

$$\frac{d\tilde{\pi}^e}{d\tau} = F_\pi(\pi^e, y^e) \equiv G_1(G_2(\pi^e, y^e), y^e) - \pi^e.$$

From (27) we have

$$\frac{d\tilde{\pi}^e}{d\tau} = \varpi^{-1} \frac{d\pi^e}{d\tau},$$

and in terms of blended expectations the second E-stability equation becomes

$$\frac{d\pi^e}{d\tau} = \varpi F_\pi(\pi^e, y^e) \equiv \varpi (G_1(G_2(\pi^e, y^e), y^e) - \pi^e).$$

These considerations imply that the earlier analysis is unchanged if the relevant state space is thought to be in terms of blended π^e where the ODE for π^e is the usual ODE for inflation expectations with the right-hand side multiplied by the weight ϖ .

⁴⁶As in Section 3.2 we set to $\Xi_t^e \equiv 1$.

(Note that the state space is the usual one when $\varpi = 1$.) Changes in ϖ correspond to changes in the adjustment speed of inflation expectations π^e , so that smaller ϖ means lower value for derivative and slower adjustment. The steady states and their E-stability properties are clearly unchanged, so we have the result:

Proposition 2. (i) *The targeted steady state is E-stable provided ϕ_y is not too large.* (ii) *The steady state (π_L, y_L) is not E-stable provided ϕ_y is not too large.* (iii) *The steady state (π_S, y_S) is E-stable.*

The proof is a straightforward modification of the proof of Proposition 1.

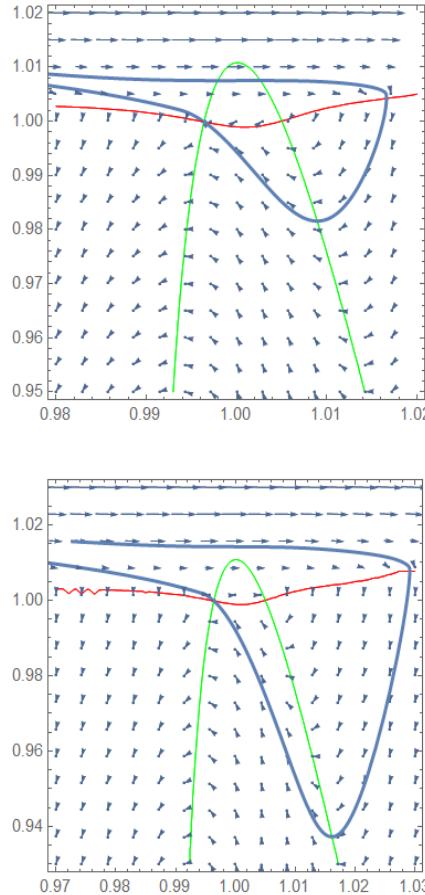


Figure 7: The top panel shows the domain of attraction of the target steady state with $\varpi = 0.8$. The bottom panel shows the domain of attraction of the target steady state with $\varpi = 0.5$. π^e is on the horizontal and y^e on vertical axis.

Looking at the global picture, the qualitative dynamics for different $\varpi \in (0, 1)$ are

unchanged relative to those in Figure 4, which corresponds to $\varpi = 1$. There is, however, a major quantitative change: the domain of attraction becomes larger when the weight $1 - \varpi$ on the fixed central bank forecast π^* is larger. See the two panels in Figure 7 which should be compared to Figure 4.

One way to see the quantitative significance of the value of ϖ is to consider the value of y^e which is on the lower boundary of the domain of attraction when $\pi^e = \pi^* = 1.005$. In Figure 4 this value is approximately $y^e = 0.98792$. In the top panel of Figure 7, with $\varpi = 0.8$, the corresponding value is $y^e \approx 0.985$, whereas in the bottom panel of Figure 7, with $\varpi = 0.5$, this value falls to $y^e \approx 0.975$. The enlargement of the domain of attraction is also very visible for high values $\pi^e >> \pi^*$.

This result has a natural interpretation. $1 - \varpi$ can be viewed as a measure of the Central Bank's credibility in being able to deliver inflation rates in line with its announced target. For $1 - \varpi$ large π^e will stay near π^* even if econometric forecasts based on recent past data give, for example, a much lower forecast. This credibility increases the robustness of the targeted steady state by increasing its domain of attraction under adaptive learning.

Again, the qualitative aspects of dynamics shown in Figure 4 remain unchanged in the two panels of Figure 7 – the possibility of a stagnation trap remains for y^e, π^e sufficiently pessimistic. A natural extension of the blended expectations approach is reinforcement learning, in which the weight ϖ is made time-varying with ϖ_t evolving based on the relative accuracy of the two forecast rules.⁴⁷ Reinforcement learning would limit the degree to which credibility could be maintained if inflation were persistently different from the target. Nonetheless, it is clear that a credible inflation target makes the targeted steady state more robust to expectation shocks under adaptive learning.

6 Discussion and Conclusions

Sluggish real economic performance and a long-lasting ZLB have made the possibility of secular stagnation a prominent topic of economic discussion. Japan has mostly been in this situation for over 20 years and the western economies US, Euro Area and UK for much of the post-2007 period. Our first objective in this paper was to extend a standard NK model in a way that exhibits stagnation as a well-defined regime for the economy, which is present despite the existence of a locally stable regime that includes the steady state targeted by policymakers. In the stagnation regime the interest rate is effectively at the ZLB and, under unchanged policies, out-

⁴⁷For an application of this approach see Honkapohja and Mitra (2019).

put and inflation will tend to slowly decline along a sequence of temporary equilibria of the economy, determined by the evolution of expectations under adaptive learning and by the sequence of random exogenous intrinsic shocks.

Our model abstracts from exogenous technological progress, as well as population growth, so the stagnation region should be viewed as a “trap” in which output tends to fall further and further below potential output, and in which inflation and inflation expectations are declining and persistently below target. The stagnation region contains a well-defined steady state – a theoretical lower bound on economic activity accompanied by rapid deflation – which acts as an attractor within the stagnation region. Reaching it would require a very passive and unresponsive government even as economic activity gradually and continuously shrinks. Thus the stagnation steady state should be viewed, not as a plausible outcome, but as a force that, within the stagnation regime, pulls output and inflation away from the targeted steady state.

A second objective of our paper has been to consider the impact of fiscal policy, and in particular the potential for fiscal policy to avoid or extract the economy from the stagnation region. In our bounded rationality setting, adaptive-learning agents take full account of the direct effects of the announced policy, but use learning rules to forecast future values of inflation and aggregate output. A large pessimistic expectation shock (due, say, to a recent financial crisis) can push the economy to the ZLB and along a path to stagnation and eventual deflation. In this setting a fiscal stimulus can be particularly potent. We have shown that a sufficiently large temporary increase in government spending can increase output and inflation enough to push the economy out of the stagnation trap. The chances of policy success are significantly greater if the policy is implemented early, before expectations deteriorate greatly. However, even if the economy is well inside the stagnation region, a large temporary fiscal stimulus can dislodge the economy from the stagnation trap so that the economy will then eventually return, under normal policy, to the targeted steady state. Combining fiscal policy with forward guidance of monetary policy can increase the probability of the economy getting out of the stagnation regime.

Our third objective has been to show the versatility of our framework for providing a range of possible macroeconomic outcomes to major shocks, resulting from the evolution of expectations driven by adaptive learning. Section 3, using a two-dimensional expectational nonstochastic nonlinear model, demonstrated that globally the two stable regions are separated by a boundary that includes a negatively-sloped curve of pessimistic output and inflation expectations, crossing a third, unstable steady state. Following a pessimistic shock to expectations, and depending on the precise size and location of the shock, there is either eventual convergence to the targeted steady state or else the economy will enter the stagnation region. For pessimistic expecta-

tions, that initially place the economy near the boundary between the regions, the economy will first travel, for a possibly extended period of time, to near the third unstable steady state, before entering one of the two regions. In this case the economy exhibits sensitive dependence on initial conditions.

A fourth objective of our paper has been to incorporate exogenous random productivity and mark-up shocks into our nonlinear set-up. To do this we make simple but natural bounded rationality assumptions – that agents use point expectations in making consumption and pricing decisions and that forecasts of key endogenous aggregates are made using linear forecast models with coefficients that are revised over time using discounted least-squares learning. Theoretically this of course implies stochastic paths, since realizations and expectations now reflect the impact of the random sequence of shocks on outcomes and on learning dynamics. This stochastic feature is illustrated using stochastic simulations, and is used in our examination of policy to summarize the effectiveness (in terms of the probability for escaping stagnation) of a temporary fiscal stimulus, for alternative sizes and lengths, in returning the economy to the targeted steady state, from pessimistic expectations of different magnitudes for which stagnation would almost always result under unchanged policy.

The extensions to the model examined in Section 5 refine the policy implications and show that our framework has the potential to accommodate a large range of economic outcomes following a major shock to expectations. For very severe negative output expectation shocks, a large fiscal stimulus for an appropriate length is needed to have a high probability of success. Forward guidance of zero interest rates for an extended period, combined with a fiscal stimulus, can increase the probability of success. In some cases forward guidance alone is insufficient, even if the guidance is for very long periods, but combined policy can be successful. Delays in implementing the policy can substantially reduce the likelihood of success and the speed with which the economy returns to the targeted steady state.

For standard calibrations of the model the economy has mild deflation at the unstable steady state, and hence if output expectations are at the targeted steady-state value then expectations of deflation would be needed to enter the stagnation trap. However, if the discount factor is near one and if credit frictions are included this critical inflation rate can be a low positive inflation rate. Thus positive but low and declining inflation expectations, as well as low output expectations, raise the possibility of the economy entering a stagnation trap. Credit frictions also reduce the size of the domain of attraction of the targeted steady state, rendering the economy less robust to expectation shocks. This can be partially offset by adopting a higher inflation target. In our last extension, we considered the impact of anchored inflation expectations associated with a credible central bank inflation target. To the extent

that achieving the target is considered credible by agents, this makes the targeted steady state quantitatively more robust, but the central qualitative features of the model remain unchanged.

From these observations it can be seen that the framework of this paper can encompass a wide range of outcomes arising from a large pessimistic shock to expectations. These results have been obtained through simple extensions of the basic standard New Keynesian model; this facilitates understanding of the key forces at work. Using this framework to explain recent (as well as future) events, such as the different responses of major economies in the wake of the 2007-9 financial crisis, is reserved for future research. For serious applied work it would, of course, be important to incorporate many features found in more elaborate models, including investment and capital, separate wage and price frictions, habit persistence, distortionary taxes, an explicit financial sector, and a fraction of hand-to-mouth households.

Appendices

A Derivations of model equations, section 2

Consumption decisions: Ricardian households are assumed to internalize the intertemporal budget constraint (IBC) of the government. The flow budget constraint of the government is

$$b_t + m_t + \Upsilon_t = g_t + m_{t-1}\pi_t^{-1} + r_t b_{t-1},$$

where we now write $r_t = R_{t-1}\pi_t^{-1}$. Setting

$$\Delta_t = g_t - \Upsilon_t - m_t + m_{t-1}\pi_t^{-1}$$

we have

$$b_t = \Delta_t + r_t b_{t-1}.$$

Note that $\Upsilon_t + m_t - m_{t-1}\pi_t^{-1}$ is total tax revenue, equal to the sum of lump-sum taxes and seigniorage.

Substituting in recursively we obtain

$$0 = r_t b_{t-1} + \sum_{j=1}^s D_{t,t+j}^{-1} \Delta_{t+j} + \Delta_t - D_{t,t+s}^{-1} b_{t+s},$$

where

$$D_{t,t+j} = \prod_{i=1}^j r_{t+i}.$$

The IBC of the government is obtained by imposing $\lim_{s \rightarrow \infty} D_{t,t+s}^{-1} b_{t+s} = 0$, which gives

$$0 = r_t b_{t-1} + \sum_{j=0}^{\infty} D_{t,t+j}^{-1} \Delta_{t+j},$$

where for convenience we set $D_{t,t} = 1$.

For households the flow budget constraint

$$c_{t,i} + m_{t,i} + b_{t,i} + \Upsilon_{t,i} = m_{t-1,i}\pi_t^{-1} + R_{t-1}\pi_t^{-1}b_{t-1,i} + \frac{P_{t,i}}{P_t}y_{t,i}$$

can be written as

$$b_{t,i} = \Lambda_{t,i} + r_t b_{t-1,i}, \text{ where}$$

$$\Lambda_{t,i} = \frac{P_{t,i}}{P_t} y_{t,i} - \Upsilon_{t,i} - c_{t,i} - m_{t,i} + m_{t-1,i} \pi_t^{-1}.$$

Hence $0 = r_t b_{t-1,i} + \sum_{j=0}^s D_{t,t+j}^{-1} \Lambda_{t+j,i} - D_{t,t+s}^{-1} b_{t+s,i}$ and imposing $\lim_{s \rightarrow \infty} D_{t,t+s}^{-1} b_{t+s,i} = 0$ gives the household IBC

$$0 = r_t b_{t-1,i} + \sum_{j=0}^{\infty} D_{t,t+j}^{-1} \Lambda_{t+j,i}.$$

We have representative agents and assume they believe future lump-sum taxes and seigniorage revenue provided to the government will be identical across agents, so that $\Upsilon_{t,i} - m_{t,i} + m_{t-1,i} \pi_t^{-1} = \Upsilon_t - m_t + m_{t-1} \pi_t^{-1}$ and $\Lambda_{t,i} = \frac{P_{t,i}}{P_t} y_{t,i} - \Upsilon_t - c_{t,i} - m_t + m_{t-1} \pi_t^{-1}$. It follows that

$$\Lambda_{t+j,i} = \frac{P_{t+j,i}}{P_{t+j}} y_{t+j,i} - c_{t+j,i} - g_{t+j} + \Delta_{t+j}.$$

Incorporating the government IBC into the household IBC yields the Ricardian household IBC, which we assume holds in expectation, i.e.

$$0 = \sum_{j=0}^{\infty} \hat{E}_{t,i} D_{t,t+j}^{-1} \left(\frac{P_{t+j,i}}{P_{t+j}} y_{t+j,i} - c_{t+j,i} - g_{t+j} \right),$$

which with point expectations becomes

$$0 = \sum_{j=0}^{\infty} D_{t,t+j}^{e-1} \left(\frac{P_{t+j,i}^e}{P_{t+j}^e} y_{t+j,i}^e - c_{t+j,i}^e - g_{t+j}^e \right).$$

Finally, to obtain the household consumption function we make use of their consumption Euler equation

$$(c_{t,i} + \xi g_t)^{-1} = \beta \hat{E}_{t,i} (r_{t+1} (c_{t+1,i} + \xi g_{t+1})^{-1}).$$

Iterating and assuming point expectations gives

$$c_{t+j,i}^e = -\xi g_{t+j,i}^e + \beta^s (D_{t,t+j}^e) (c_{t,i} + \xi g_t).$$

Substituting for $c_{t+j,i}^e$ in the household IBC and solving for c_t gives the consumption

function

$$\begin{aligned} c_{t,i} &= (1 - \beta) \left[\frac{P_{t,i}}{P_t} y_{t,i} - g_t \left(1 + \frac{\xi\beta}{1 - \beta} \right) \right] + \\ &\quad (1 - \beta) \sum_{s=1}^{\infty} (D_{t,t+s,i}^e)^{-1} \left[\left(\frac{P_{t+s,i}}{P_{t+s}} \right)^e y_{t+s,i}^e - g_{t+s,i}^e (1 - \xi) \right]. \end{aligned}$$

Imposing the non-negativity constraint $c_{t,i} \geq 0$ gives (9) in the main text.

Remarks: Seigniorage reduces the required explicit tax Υ_t on households, but this is offset by the cost to households of holding real balances. In the Ricardian case, with monetary policy specified as an interest-rate rule, it is unnecessary to track money supply and demand. However, it is straightforward to show that with our utility function real money demand satisfies $m_{t,i} = \chi\beta (1 - R_t^{-1})^{-1} c_{t,i}$. The cashless limit corresponds to $\chi \rightarrow 0$.

Production decisions: The adjustment cost function $\Phi(\frac{P_{t,j}}{P_{t-1,j}})$ is the Linex function

$$\Phi \left(\frac{P_{t,i}}{P_{t-1,i}} \right) \equiv \phi \left[\frac{\exp(-\psi(P_{t,i}/P_{t-1,i} - \pi^*)) + \psi(P_{t,i}/P_{t-1,i} - \pi^*) - 1}{\psi^2} \right].$$

We compute the derivative

$$\frac{d}{dP_{t,i}} \left[\Phi \left(\frac{P_{t,i}}{P_{t-1,i}} \right) \right] = \frac{\phi}{\psi} P_{t-1,i}^{-1} [-\exp(-\psi(P_{t,i}/P_{t-1,i} - \pi^*)) + 1].$$

Note that

$$\Phi'(\pi_i) = \frac{\phi}{\psi} (-\exp(-\psi(\pi_i - \pi^*)) + 1),$$

so

$$\frac{d}{dP_{t,i}} \left[\Phi \left(\frac{P_{t,i}}{P_{t-1,i}} \right) \right] = P_{t-1,i}^{-1} \Phi' \left(\frac{P_{t,i}}{P_{t-1,i}} \right).$$

The first-order condition for optimal price setting is

$$\begin{aligned} 0 &= \frac{\partial U_{t,i}}{\partial P_{t,i}} + \beta E_{t,i} \frac{\partial U_{t+1,i}}{\partial P_{t,i}} = \frac{\nu_t}{\alpha} h_{t,i}^{\varepsilon+1} \frac{1}{P_{t,i}} - \Phi'(\pi_{t,i}) \frac{1}{P_{t-1,i}} \\ &\quad + (c_{t,i} + \xi g_t)^{-1} (1 - \nu_t) y_t \left(\frac{P_{t,i}}{P_t} \right)^{-\nu_t} \frac{1}{P_t} + \beta \Phi'(\pi_{t+1,i}^e) \left(\frac{P_{t+1,i}}{P_{t,i}^2} \right)^e, \end{aligned}$$

where again we have used point expectations and here $\pi_{t,i} = P_{t,i}/P_{t-1,i}$. Multiplying

the right-hand side by $P_{t,i}$ we can write this equation as

$$\Phi'(\pi_{t,i})\pi_{t,i} = \frac{\nu_t}{\alpha}h_{t,i}^{\varepsilon+1} + (c_{t,i} + \xi g_t)^{-1}(1 - \nu_t)y_t \left(\frac{P_{t,i}}{P_t} \right)^{1-\nu_t} + \beta\Phi'(\pi_{t+1,i}^e)\pi_{t+1,i}^e.$$

We now discuss the properties of

$$\Phi'(\pi)\pi = \frac{\phi}{\psi}\pi(-\exp(-\psi(\pi - \pi^*)) + 1).$$

The function $\Phi'(\pi)\pi$ is monotonically increasing above a critical value $\check{\pi}$ which is given by the condition

$$\frac{d}{d\pi}\Phi'(\pi)\pi = 0. \quad (28)$$

We compute the derivative

$$\frac{d}{d\pi}\Phi'(\pi)\pi = \frac{\phi}{\psi}(1 - (1 - \phi\pi)\exp(-\psi(\pi - \pi^*))),$$

so the condition giving $\check{\pi}$ can be written as

$$1 = (1 - \phi\pi)\exp(-\psi(\pi - \pi^*)).$$

This equation has a unique solution $\check{\pi} < \phi^{-1}$. It is easily seen that (i) $\check{\pi}$ is increasing in ψ with $\lim_{\psi \rightarrow \infty} \check{\pi} = 1/\phi^{-1}$ and (ii) $\check{\pi}$ is decreasing in ϕ with $\lim_{\phi \rightarrow \infty} \check{\pi} = 0$ *ceteris paribus*. Throughout the paper we restrict attention to regions for which $\pi > \check{\pi}$. In the calibrated model we will compute $\check{\pi}$ to check and impose the inequality $\pi > \check{\pi}$ when solving for the temporary equilibrium.

Using the production function, (10) in the main text is

$$\begin{aligned} \zeta_{t,i} &= \frac{\nu_t}{\alpha}h_{t,i}^{\varepsilon+1} - (\nu_t - 1)(c_{t,i} + \xi g_t)^{-1}y_t \left(\frac{P_{t,i}}{P_t} \right)^{1-\nu_t} \\ &= \frac{\nu_t}{\alpha} \left(\frac{y_{t,i}}{A_t} \right)^{(1+\varepsilon)/\alpha} - (\nu_t - 1)(c_{t,i} + \xi g_t)^{-1}y_t \left(\frac{P_{t,i}}{P_t} \right)^{1-\nu_t}. \end{aligned}$$

Here

$$\frac{y_{t,i}}{A_t} = \frac{\int_0^1 c_{t,j}(i) dj + g_t(i)}{A_t} = \frac{c_t(i) + g_t(i)}{A_t}$$

is the total demand for variety i .

Note that the term $y_t \left(\frac{P_{t,i}}{P_t} \right)^{1-\nu_t}$ combines y_t , which is exogenous to the firm, with the relative price $\frac{P_{t,i}}{P_t}$, in which the aggregate price level is exogenous while $P_{t,i}$ is a decision variable of the firm. Iterating forward we get the expression (11)

$$\Phi'(\pi_{t,i})\pi_{t,i} = \zeta_{t,i} + \sum_{s=1}^{\infty} \beta^s \zeta_{t+s,i}^e,$$

which is our infinite-horizon pricing decision rule. Here $\zeta_{t+s,i}^e$ is the point expectation of

$$\zeta_{t+s,i} = \frac{\nu_{t+s}}{\alpha} \left(\frac{y_{t+s,i}}{A_{t+s}} \right)^{(1+\varepsilon)/\alpha} - (\nu_{t+s} - 1) y_{t+s} \left(\frac{P_{t+s,i}}{P_{t+s}} \right)^{1-\nu_t} \times (c_{t+s,i} + \xi g_{t+s})^{-1},$$

where

$$y_{t+s,i} = c_{t+s}(i) + g_{t+s}(i)$$

is the future market demand for variety i .

B Proofs of propositions

We start by computing the partial derivatives of the right-hand sides of differential equations (22)-(24):

$$\frac{\partial F_\pi}{\partial \pi^e} = D_y \tilde{G}_1 D_{\pi^e} \tilde{G}_2 - 1, \quad \frac{\partial F_\pi}{\partial \Xi^e} = D_{\Xi^e} \tilde{G}_1 + D_y \tilde{G}_1 D_{\Xi^e} \tilde{G}_2, \quad \frac{\partial F_\pi}{\partial y^e} = D_{y^e} \tilde{G}_1 + D_y \tilde{G}_1 D_{y^e} \tilde{G}_2$$

and

$$\frac{\partial F_y}{\partial \pi^e} = D_{\pi^e} \tilde{G}_2, \quad \frac{\partial F_y}{\partial \Xi^e} = D_{\Xi^e} \tilde{G}_2, \quad \frac{\partial F_y}{\partial y^e} = D_{y^e} \tilde{G}_2 - 1.$$

Note that there is no dependence of actual relative price on expectations. Then write the E-stability differential equation in vector form as

$$\begin{pmatrix} \frac{\partial \pi^e}{\partial \tau^e} \\ \frac{\partial \Xi^e}{\partial \tau^e} \\ \frac{\partial y^e}{\partial \tau} \end{pmatrix} = \begin{pmatrix} F_\pi(\pi^e, \Xi^e, y^e) \\ 1 - \Xi^e \\ F_y(\pi^e, \Xi^e, y^e) \end{pmatrix},$$

where $F_y(\cdot)$ and $F_\pi(\cdot)$ are given (23) and (24). We get the Jacobian

$$DFI = \begin{pmatrix} D_y \tilde{G}_1 D_{\pi^e} \tilde{G}_2 - 1 & D_{\Xi^e} \tilde{G}_1 + D_y \tilde{G}_1 D_{\Xi^e} \tilde{G}_2 & D_{y^e} \tilde{G}_1 + D_y \tilde{G}_1 D_{y^e} \tilde{G}_2 \\ 0 & -1 & 0 \\ D_{\pi^e} \tilde{G}_2 & D_{\Xi^e} \tilde{G}_2 & D_{y^e} \tilde{G}_2 - 1 \end{pmatrix}.$$

Proof of Proposition 1: (a) (i) Consider the case $\phi_y = 0$. Calculating the derivatives of the Jacobian at the target steady state we get

$$\begin{aligned} (Q^{-1})' &= (\Phi''\pi + \Phi')^{-1} = (\phi\pi^*)^{-1} > 0 \text{ so} \\ D_y \tilde{G}_1 &= (Q^{-1})' \left(\frac{\nu(1+\varepsilon)}{\alpha^2} (y^*/A)^{(1+\varepsilon)/\alpha-1} + \frac{(\nu-1)(1-\xi)g}{(y^* - (1-\xi)g)^2} \right) > 0. \\ D_{\pi^e} \tilde{G}_2 &= (\beta^{-1} - 1)(y^* - g(1-\xi)) \left(\frac{R(\pi_t^e, y_t^e) - \pi_t^e D_\pi R(\pi_t^e, y_t^e)}{(R(\pi_t^e, y_t^e) - \pi_t^e)^2} \right) < 0. \end{aligned}$$

as

$$\begin{aligned} D_\pi R(\pi, y) &= \frac{BR^*}{\pi^*} \left(\frac{\pi}{\pi^*} \right)^{(R^*(B-1)+1)/(R^*-1)} \left(\frac{y}{y^*} \right)^{\phi_y} \\ &= B\beta^{-1} \left(\frac{\pi}{\pi^*} \right)^{(R^*(B-1)+1)/(R^*-1)} \left(\frac{y}{y^*} \right)^{\phi_y} \end{aligned}$$

and $R(\pi^*, y^*) - \pi^* D_\pi R(\pi^*, y^*) = \beta^{-1}\pi^*(1-B) < 0$ at the target steady state. Also

$$\begin{aligned} D_{\Xi^e} \tilde{G}_1 &= (Q^{-1})' \left[\frac{-\beta}{1-\beta} (\nu-1) y^* \times (y^* - (1-\xi)g)^{-1} \right] < 0 \\ D_{\Xi^e} \tilde{G}_2 &= (y^* - g(1-\xi)) > 0 \end{aligned}$$

$$\begin{aligned} D_{y^e} \tilde{G}_1 &= (Q^{-1})' \frac{\beta}{1-\beta} \left[\frac{\nu(1+\varepsilon)}{\alpha^2} (y^*/A)^{(1+\varepsilon-\alpha)/\alpha} + \frac{(\nu-1)(1-\xi)g}{(y^* - (1-\xi)g)^2} \right] > 0 \\ D_{y^e} \tilde{G}_2 &= 1 \end{aligned}$$

if $\phi_y = 0$. So in this case we get

$$DFI = \begin{pmatrix} - & ? & + \\ 0 & -1 & 0 \\ - & + & 0 \end{pmatrix}$$

Looking at this matrix, it is seen that the dynamics of Ξ^e is independent from other variables and is convergent. Then the remaining 2×2 matrix

$$\begin{pmatrix} - & + \\ - & 0 \end{pmatrix}$$

has negative trace and positive determinant and is thus a stable matrix. The result follows by continuity of eigenvalues.

If ϕ_y is not zero, we have

$$D_{y^e}\tilde{G}_2 = 1 + (\beta^{-1} - 1) \left[(1 - g(1 - \xi)) \left(\frac{\pi}{D_y R(\pi, y) - \pi} \right) \right].$$

For the interest rate rule we get

$$D_y R(\pi, y) = (R^* - 1) \left(\frac{\pi}{\pi^*} \right)^{(R^* B / (R^* - 1)} \frac{\phi_y}{y^*} \left(\frac{y}{y^*} \right)^{\phi_y - 1},$$

so

$$D_y R(\pi^*, y^*) = (R^* - 1) \phi_y / y^*$$

at the target steady state and thus

$$D_{y^e}\tilde{G}_2 = 1 + (\beta^{-1} - 1) \left[(1 - g(1 - \xi)) \left(\frac{\pi^*}{(R^* - 1)\phi_y / y^* - \pi^*} \right) \right].$$

Now $R^* - 1 \gtrapprox 0$ is small, something like 0.02, while $\phi_y \approx 8$ and $y^* \approx 1$, whereas $\pi^* \gtrapprox 1$. Then $D_{y^e}\tilde{G}_2 < 1$ and the targeted steady state is E-stable.

(ii) Doing calculations similar to above, set first $\phi_y = 0$ and we get

$$(Q^{-1})' = \left(\frac{\phi(1 - (1 - \psi\pi_L) \exp(\pi_L - \pi^*))}{\psi} \right)^{-1} > 0 \text{ normally so}$$

$$D_y \tilde{G}_1 = (Q^{-1})' \left(\frac{\nu(1 + \varepsilon)}{\alpha^2} (y_L/A)^{1+\varepsilon-\alpha}/\alpha + \frac{(\nu - 1)(1 - \xi)g}{(y_L - (1 - \xi)g)^2} \right) > 0.$$

$$D_{\pi^e}\tilde{G}_2 = (\beta^{-1} - 1)(y_L - g(1 - \xi)) \left(\frac{R(\pi_L, y_L) - \pi_L D_\pi R(\pi_L, y_L)}{(R(\pi_L, y_L) - \pi_L)^2} \right) < 0,$$

as

$$1 + (R^* - 1) \left(\frac{\pi_L}{\pi^*} \right)^{BR^*/(R^*-1)} \left(\frac{y_L}{y^*} \right)^{\phi_y} < B\beta^{-1}\pi^* \left(\frac{\pi_L}{\pi^*} \right)^{BR^*/(R^*-1)} \left(\frac{y_L}{y^*} \right)^{\phi_y} \text{ normally.}$$

Also

$$\begin{aligned} D_{\Xi^e} \tilde{G}_1 &= (Q^{-1})' \left[\frac{-\beta}{1-\beta} (\nu - 1) y_L \times (y_L - (1-\xi)g)^{-1} \right] < 0 \\ D_{\Xi^e} \tilde{G}_2 &= (y_L - g(1-\xi)) > 0 \\ D_{y^e} \tilde{G}_1 &= (Q^{-1})' \frac{\beta}{1-\beta} \left[\frac{\nu(1+\varepsilon)}{\alpha^2} (y_L/A)^{(1+\varepsilon-\alpha)/\alpha} + \frac{(\nu-1)(1-\xi)g}{(y_L - (1-\xi)g)^2} \right] > 0 \\ D_{y^e} \tilde{G}_2 &= 1 - \frac{\phi_y(y_L/y^*)(\beta^{-1}\pi_L - 1)}{(\beta^{-1} - 1)\pi_L}. \end{aligned}$$

Normally, $D_{y^e} \tilde{G}_1 + D_y \tilde{G}_1 D_{y^e} \tilde{G}_2 > 0$ and considering the case $\phi_y = 0$, we get

$$DFI = \begin{pmatrix} ? & ? & + \\ 0 & -1 & 0 \\ + & + & 0 \end{pmatrix}.$$

Considering the 2×2 matrix

$$\begin{pmatrix} ? & + \\ + & 0 \end{pmatrix}$$

it is seen that the determinant is negative, so the (π_L, y_L) is not E-stable.

The case $\phi_y > 0$ but not too large also leads to instability depending on the parameter values. This is true in numerical analyses where $\phi_y = 8.25$.

(iii) At the stagnation steady state $y_t = \tilde{G}_2(\pi_t^e, \Xi_t^e, y_t^e)$ has to be locally constant, so the following partial derivatives are zero $D_{\pi^e} \tilde{G}_2 = D_{\Xi^e} \tilde{G}_2 = D_{y^e} \tilde{G}_2 = 0$. Thus the Jacobian matrix becomes

$$DFI = \begin{pmatrix} -1 & D_{\Xi^e} \tilde{G}_1 & D_{y^e} \tilde{G}_1 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix},$$

where

$$D_{\Xi^e} \tilde{G}_1 = (Q^{-1})' \left[\frac{-\beta}{1-\beta} (\nu - 1) \xi^{-1} \right] < 0$$

$$D_{y^e} \tilde{G}_1 = (Q^{-1})' \frac{\beta}{1-\beta} \left[\frac{\nu(1+\varepsilon)}{\alpha^2} (g)^{1+\varepsilon-\alpha}/\alpha + \frac{(\nu-1)(1-\xi)g}{(\xi g)^2} \right] > 0$$

So we get

$$DGI = \begin{pmatrix} -1 & - & + \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

Eliminating the independent dynamics if Ξ^e , we get the matrix

$$\begin{pmatrix} -1 & + \\ 0 & -1 \end{pmatrix}$$

which yields stability.

(b). In non-stochastic models, with constant-gain, steady-state learning is locally stable for sufficiently small gains if and only if E-stability holds. We provide a sketch of the proof. Consider a linear(-ized) model $y_t = Ty_t^e$, assuming zero intercept without loss of generality, and the constant-gain rule $a_t = a_{t-1} + \gamma(y_t - a_{t-1})$, where $0 < \gamma < 1$. The PLM is $y_t^e = a_{t-1}$. Then $a_t = a_{t-1} + \gamma(Ta_{t-1} - a_{t-1})$ and the system is convergent if matrix $\gamma T + (1-\gamma)I$ has eigenvalues t_i inside the unit circle; equivalently $T + \gamma^{-1}(1-\gamma)I$ has eigenvalues inside the circle with radius γ^{-1} . Eigenvalues of the latter matrix are equal to $t_i + \gamma^{-1}(1-\gamma)$, so the t_i 's must lie inside unit circle with origin at $(1 - \gamma^{-1}, 0)$ and radius γ^{-1} . Letting $\gamma \rightarrow 0$ yields the E-stability condition that real parts of t_i must be less than 1. ■

C Implementation of stochastic model

From Section 2 we have the representative agent NK PC temporary equilibrium (TE) equation

$$Q(\pi_t) = \zeta_t + \sum_{s=1}^{\infty} \beta^s \zeta_{t+s}^e, \text{ where } Q(\pi_t) = \Phi'(\pi_t)\pi_t, \quad (29)$$

with $Q(\pi_t) > 0$ for $\pi_t > \check{\pi}$, and where

$$\zeta_t = \frac{\nu_t}{\alpha} \left(\frac{y_t}{A_t} \right)^{(1+\varepsilon)/\alpha} - (\nu_t - 1) y_t \times (y_t - (1 - \xi) g_t)^{-1}$$

and

$$\zeta_{t+s}^e = \frac{\nu_t}{\alpha} \left(\frac{y_{t+s}^e}{A_{t+s}^e} \right)^{(1+\varepsilon)/\alpha} - (\nu_{t+s}^e - 1) y_{t+s}^e \times (y_{t+s}^e - (1 - \xi) g_{t+s})^{-1}, \quad (30)$$

for $s = 1, 2, 3, \dots, T$. As we discuss below, we will set ζ_{t+s}^e at its perceived mean value after $T + 1$ periods, for some (suitably large) period T . Here we are assuming $\Xi_t^e \equiv 1$ on the grounds that agents have learnt that in fact they will decide to set the same price as other agents. Note that ν_t is stochastic and the we assume that, faced with stochastic shocks in a nonlinear setting, agents use point expectations. We also assume that the future path of government spending is credibly announced and implemented; hence we have assumed that $g_{t+s}^e = g_{t+s}$.

Also from Section 2, the NK AD temporary equilibrium equation is obtained by combining the IH consumption function, market clearing, i.e. $y_t = c_t + g_t$, and $\Xi_t^e \equiv 1$. This yields the TE equation for output

$$y_t = (1 - \xi) g_t + (\beta^{-1} - 1) \sum_{s=1}^{\infty} (D_{t,t+s}^e)^{-1} (y_{t+s}^e - (1 - \xi) g_{t+s}^e), \quad (31)$$

where

$$D_{t,t+s}^e = \prod_{j=1}^s r_{t+j}^e \text{ and } r_{t+j}^e = \frac{R_{t+j-1}^e}{\pi_{t+j}^e}.$$

Here for $j = 1$ we have $R_t^e = R_t$.⁴⁸ The interest-rate rule is assumed known and is given by a forward-looking rule (16). Again we assume point expectations for forecasting all unknown future values.

We turn next to the data-generating process for the stochastic shocks. As assumed in Section 3.3, $\ln A_t$ and $\ln \nu_t$ are independent stationary exogenous AR(1) processes

$$\ln(A_{t+1}/\bar{A}) = \rho_A \ln(A_t/\bar{A}) + \ln \varepsilon_{A,t+1}$$

⁴⁸As we explain below we need to replace $r_{t+j}^e = R_{t+j-1}^e / \pi_{t+j}^e$ by $r_{t+j}^e = \beta^{-1}$ for $j \geq T_1$ for some positive T_1 .

where $0 \leq \rho_A < 1$, $\ln \varepsilon_{A,t} \stackrel{iid}{\sim} N(0, \sigma_A^2)$, and

$$\ln(\nu_{t+1}/\bar{\nu}) = \rho_\nu \ln(\nu_t/\bar{\nu}) + \ln \varepsilon_{\nu,t+1}$$

where $0 \leq \rho_\nu < 1$, $\ln \varepsilon_{\nu,t} \stackrel{iid}{\sim} N(0, \sigma_\nu^2)$.

It follows that

$$\nu_{t+1}/\bar{\nu} = (\nu_t/\bar{\nu})^{\rho_\nu} \varepsilon_{\nu,t+1} \text{ and } A_{t+1}/\bar{A} = (A_t/\bar{A})^{\rho_A} \varepsilon_{A,t+1},$$

and that

$$\nu_{t+s}/\bar{\nu} = (\nu_t/\bar{\nu})^{\rho_\nu^s} \prod_{j=0}^{s-1} \varepsilon_{\nu,t+j}^{\rho_\nu^j}.$$

Under point expectations $\ln \varepsilon_{\nu,t+j}^e = 0$ and $\varepsilon_{\nu,t+j}^e = 1$ so that

$$\nu_{t+s}^e = \bar{\nu} (\nu_t/\bar{\nu})^{\rho_\nu^s},$$

and analogously we have

$$A_{t+s}^e = \bar{A} (A_t/\bar{A})^{\rho_A^s}.$$

In Section 3.3, the PLMs for output and inflation use a linear forecasting rule based of the observed exogenous variables. To first-order these correspond to a stochastic REE at a steady state. Thus the perceived laws of motion are

$$\begin{aligned} \ln(y_t) &= f_y + d_{yA} \ln(A_t/\bar{A}) + d_{y\nu} \ln(\nu_t/\bar{\nu}) + \eta_{yt} \\ \ln(\pi_t) &= f_\pi + d_{\pi A} \ln(A_t/\bar{A}) + d_{\pi\nu} \ln(\nu_t/\bar{\nu}) + \eta_{\pi t}, \end{aligned}$$

where $\eta_{yt}, \eta_{\pi t}$ are perceived white noise shocks.

Under constant gain RLS learning the coefficients $\phi = (f_y, d_{yA}, d_{y\nu}, f_\pi, d_{\pi A}, d_{\pi\nu})$ are time-varying and updated over time using recursive least squares regressions of $(\ln(y_t), \ln(\pi_t))$ on $(1, \tilde{A}_t, \tilde{\nu}_t)$.

Letting $f_y, d_{yA}, d_{y\nu}, f_\pi, d_{\pi A}, d_{\pi\nu}$ now denote the time t values of their estimates, expectations of output s steps ahead are given by

$$y_{t+s}^e = e^{f_y} \tilde{A}_t^{\rho_A^s d_{yA}} \tilde{\nu}_t^{\rho_\nu^s d_{y\nu}} \text{ for } s = 1, \dots, T,$$

where as usual point expectations are assumed. Here T denotes the period after which agents believe that all relevant processes will have reverted to their mean steady-state values. Thus

$$y_{t+s}^e = e^{f_y}, \nu_{t+s}^e = \bar{\nu}, A_{t+s}^e = \bar{A} \text{ and } g_{t+s} = \bar{g} \text{ for } s \geq T + 1.$$

Here \bar{g} is the original level of g to which g_t reverts after the fiscal stimulus, and we assume $t + T + 1 > T_p$, where T_p is the length of the fiscal stimulus. Similarly for ζ_{t+s}^e with $s \geq T + 1$ we replace (30) with

$$\zeta_{t+s}^e = \bar{\zeta} \equiv \frac{\bar{\nu}}{\alpha} \left(\frac{e^{f_y}}{\bar{A}} \right)^{(1+\varepsilon)/\alpha} - (\bar{\nu} - 1) e^{f_y} \times (e^{f_y} - (1 - \xi) \bar{g})^{-1} \text{ for } s \geq T.$$

Using these expectations π_t is determined by the temporary equilibrium equation

$$Q(\pi_t) = \zeta_t + \sum_{s=1}^T \beta^s \zeta_{t+s}^e + \frac{\beta^{T+1}}{1-\beta} \bar{\zeta}. \quad (32)$$

We now turn to the temporary equilibrium AD equation. Expectations y_{t+s}^e and π_{t+j}^e are given as above. For the discount factors we have

$$D_{t,t+s}^e = \prod_{j=1}^s \frac{R_{t+j-1}^e}{\pi_{t+j}^e}.$$

where R_{t+j-1}^e is given by the forward-looking R -rule (16). Here

$$R_t = R(\pi_{t+1}^e, y_{t+1}^e) \text{ with } R_{t+j-1}^e = R(\pi_{t+j}^e, y_{t+j}^e)$$

so that

$$D_{t,t+s}^e = \prod_{j=1}^s \frac{R(\pi_{t+j}^e, y_{t+j}^e)}{\pi_{t+j}^e}, \text{ for } s \leq T_1 - 1.$$

The restriction $s \leq T_1 - 1$ is included because in order to ensure that consumption and output is positive and finite we need discount factors $D_{t,t+s}^e$ to be bounded above 1. This can be an issue because for some π^e between π_L and π^* , and for a range of y^e if the rule also depends on y^e , the interest rate $R(\pi^e, y^e)$ can be less than π^e . We avoid this difficulty by assuming that after T_1 periods the expected real interest rate factor is the steady-state value β^{-1} . Thus we assume

$$r_{t+j}^e = \beta^{-1} \text{ for } j \geq T_1$$

which implies that

$$D_{t,t+s}^e = \prod_{j=1}^{T_1-1} \frac{R(\pi_{t+j}^e, y_{t+j}^e)}{\pi_{t+j}^e} \beta^{-(s-(T_1-1))}, \text{ for } s \geq T_1.$$

We assume that the ‘‘truncation’’ parameter $T > T_1, T_p$.

Incorporating the assumption that expectations y^e, π^e return to their perceived steady-state values after T periods, we arrive at the AD temporary equilibrium equations

$$\begin{aligned} y_t &= (1 - \xi) g_t + (\beta^{-1} - 1) \sum_{s=1}^T (D_{t,t+s}^e)^{-1} (y_{t+s}^e - (1 - \xi) g_{t+s}^e) + \quad (33) \\ &\quad (\beta^{-1} - 1) (e^{f_y} - \bar{g}(1 - \xi)) \sum_{s=T+1}^{\infty} (D_{t,t+s}^e)^{-1}, \text{ where} \\ &\quad \sum_{s=T+1}^{\infty} (D_{t,t+s}^e)^{-1} = (D_{t,t+T_1-1}^e)^{-1} (1 - \beta)^{-1} \beta^{T-T_1+2}. \end{aligned}$$

The latter equation is obtained from

$$\sum_{s=T+1}^{\infty} (D_{t,t+s}^e)^{-1} = (D_{t,t+T_1-1}^e)^{-1} \sum_{s=T+1}^{\infty} \beta^{s-T_1+1}.$$

The forward-looking R -rule has the advantage that π_t, y_t can be solved explicitly using the above equations (32) and (33). As already noted, the results for the contemporaneous rule are similar.

D Calibration details

The parameter values in the main text are chosen as follows. $\alpha = 0.7$, $\beta = 0.99$ and $\varepsilon = 1$ are standard. There are various suggestions for ξ and we set $\xi = 0.4$. The frequency of price change is that $1/3 (= 1 - \eta)$ of firms change prices per quarter. This is consistent with Nakamura and Steinsson (2008) and Kehoe and Midrigan (2015). Various estimates of ν or of the markup $\nu/(\nu - 1)$ have been used with estimates of ν ranging from 21 to 3.5. Keen and Wang (2007) give the relation between these parameters and the Rotemberg quadratic adjustment cost parameter

$$\gamma = \frac{(\nu - 1)\eta}{(1 - \eta)(1 - \beta\eta)}.$$

We choose $\nu = 13.5$, corresponding to a markup of about 8% and $\eta = 0.67$ which gives $\gamma = 75$.

For Linex adjustment cost functions the parameter estimates for ϕ, ψ vary widely. Note that $\phi \rightarrow \gamma$ as $\psi \rightarrow 0$. In most papers adjustment costs are assumed to be proportional to output or profit, whereas we use a non-proportional setup to avoid multiplicities. However if we normalize steady-state target output to $y = 1$ then the parameters are comparable. Also near the steady state marginal utilities drop out to first order. We choose $\phi = 75$ and $\psi = 20$, which gives a fairly close approximation to the quadratic in the range $\pi = 1.00$ to $\pi = 1.01$, i.e. 0 to 4% annual inflation.

For technology we set $\bar{A} = 1.113$, which gives a high steady state $\bar{y} \approx 1.00003 \approx 1$ with $\bar{g} = 0.2$.

For productivity and mark-up shock calibrations we set first-order autocorrelation parameters to $\rho_A = \rho_v = 0.5$ and standard deviations for the log innovations, in decimal form, to $\sigma_A = 0.0015$ and $\sigma_v = 0.0001$. Both the serial correlation and autocorrelation parameters are smaller than those found by Smets and Wouters (2007), but their estimates are for models under RE and with additional frictions. Adaptive learning dynamics add additional volatility relative to RE, particularly in purely forward-looking models.

E Data for Figures 1 and 2

Figure 1: The interest-rate rule curve takes the form $\mathcal{I} = A * \exp(B\Pi)$, where Π denotes net inflation and \mathcal{I} denotes the net interest rate. Japan switched the policy target in 2013 to monetary base.

Figure 2: Data are from Macrobond data base which in turn utilizes standard data sources. GDP data is volume data with 2010 as reference year and in local currency. GDP data is annualized. This was specifically done for the Euro area by multiplying quarterly data by 4. Population data is total population and it is interpolated for quarters.

F Calibrating the Taylor rule

To calibrate the interest rate rule

$$R_t = R(\pi_{t+1}^e, y_{t+1}^e) = 1 + (R^* - 1) \left(\frac{\pi_{t+1}^e}{\pi^*} \right)^{BR^*/(R^*-1)} \left(\frac{y_{t+1}^e}{y^*} \right)^{\phi_y}, \quad (34)$$

where y^* is output level at the target steady state, we relate (34) to the usual Taylor rule. Rearranging and taking logs we get

$$\begin{aligned} & \log(R_t - 1) - \log(R^* - 1) \\ = & \frac{BR^*}{R^* - 1} (\log \pi_{t+1}^e - \log \pi^*) + \phi_y (\log y_{t+1}^e - \log y^*). \end{aligned}$$

Multiplying by $(R^* - 1)$ and approximating log differences by percentage changes we get

$$R_t - R^* = BR^* \left(\frac{\pi_{t+1}^e - \pi^*}{\pi^*} \right) + (R^* - 1)\phi_y \left(\frac{y_{t+1}^e - y^*}{y^*} \right).$$

Thus BR^* is the inflation coefficient and $(R^* - 1)\phi_y$ is the output coefficient in the usual linear Taylor rule. Assuming a quarterly calibration one should have

$$\begin{aligned} BR^* &= 1.5 \\ (R^* - 1)\phi_y &= \frac{0.5}{4}. \end{aligned}$$

At the target steady state $R^* = \beta^{-1}\pi^*$ we get

$$\phi_y = \frac{0.5}{4}/(0.01515) \approx 8.25$$

when $\beta = 0.99$ and $\pi^* = 1.005$.

G Additional numerical results

G.1 Convergence probabilities for $y_0^e = 0.985$

Each table gives the probabilities for the specified value of T_m and ranges of values for \bar{g}' and T_p based on 100 replications.

$T_p \setminus \bar{g}'$	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8
1	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	7	57
5	0	0	0	0	0	0	0	36	60	32	4
6	0	0	0	0	0	0	54	35	2	0	0
7	0	0	0	0	0	57	5	1	0	0	0
8	0	0	0	0	55	5	1	0	0	0	0

$$y_0^e = 0.985, \text{ gain} = 0.01, T_m = 1$$

$T_p \setminus \bar{g}'$	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8
1	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	1	51	61	62
5	0	0	0	0	0	0	62	66	17	4	0
6	0	0	0	0	0	0	1	56	27	1	0
7	0	0	0	0	50	12	0	0	0	0	0
8	0	0	0	46	15	1	0	0	0	0	0

$$y_0^e = 0.985, \text{gain}=0.01, T_m = 2$$

$T_p \setminus \bar{g}'$	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8
1	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	27	55
4	0	0	0	0	0	0	35	54	66	44	10
5	0	0	0	0	0	59	62	13	1	0	0
6	0	0	0	3	55	21	2	0	0	0	0
7	0	0	0	57	19	1	1	0	0	0	0
8	0	0	21	30	0	0	0	0	0	0	0

$$y_0^e = 0.985, \text{gain}=0.01, T_m = 3$$

$T_p \setminus \bar{g}'$	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8
1	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	4
3	0	0	0	0	0	0	6	46	55	64	71
4	0	0	0	0	6	57	60	66	28	6	0
5	0	0	0	32	61	58	10	0	1	0	0
6	0	0	0	3	56	25	1	0	0	0	0
7	0	0	55	26	0	0	0	0	0	0	0
8	0	0	38	1	1	0	0	0	0	0	0

$$y_0^e = 0.985, \text{gain}=0.01, T_m = 4$$

$T_p \setminus \bar{g}'$	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8
1	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	12	43	58	62
3	0	0	0	0	15	55	61	69	52	24	8
4	0	0	0	41	65	73	27	6	0	0	0
5	0	0	34	65	43	8	1	0	0	0	0
6	0	0	58	34	3	1	0	0	0	0	0
7	0	38	48	2	1	0	0	0	0	0	0
8	0	53	0	0	0	0	0	0	0	0	0

$$y_0^e = 0.985, \text{gain}=0.01, T_m = 5$$

$T_p \setminus \bar{g}_1$	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8
1	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	37	64	60	69	67	41
3	0	0	0	45	62	73	36	12	3	1	0
4	0	0	60	69	37	6	0	1	0	0	0
5	0	32	65	30	4	0	0	0	0	0	0
6	0	64	48	4	0	0	0	0	0	0	0
7	0	57	7	1	0	0	0	0	0	0	0
8	31	36	0	0	0	0	0	0	0	0	0

$$y_0^e = 0.985, \text{gain}=0.01, T_m = 6$$

$T_p \setminus \bar{g}'$	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8
1	0	0	0	0	0	0	0	14	39	49	58
2	0	0	0	42	61	65	67	34	14	8	2
3	0	12	61	62	33	9	2	0	0	0	0
4	0	63	65	21	0	0	0	0	0	0	0
5	25	65	22	0	0	0	0	0	0	0	0
6	49	46	4	0	0	0	0	0	0	0	0
7	50	21	1	0	0	0	0	0	0	0	0
8	59	3	1	0	0	0	0	0	0	0	0

$$y_0^e = 0.985, \text{gain} = 0.01, T_m = 7$$

$T_p \setminus \bar{g}'$	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8
1	0	0	0	0	32	45	53	63	60	61	50
2	0	32	55	58	41	19	8	1	1	1	0
3	21	64	46	17	1	1	1	0	0	0	0
4	58	53	11	0	0	0	0	0	0	0	0
5	66	20	0	0	0	0	0	0	0	0	0
6	53	3	1	0	0	0	0	0	0	0	0
7	30	0	0	0	0	0	0	0	0	0	0
8	30	0	0	0	0	0	0	0	0	0	0

$$y_0^e = 0.985, \text{gain} = 0.01, T_m = 8$$

G.2 Convergence probabilities for $y_0^e = 0.98$

Each table gives the probabilities for the specified value of T_m and ranges of values for \bar{g}' and T_p with 100 replications.

$T_p \setminus \bar{g}'$	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8
1	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	38
7	0	0	0	0	0	0	0	0	29	19	2
8	0	0	0	0	0	0	0	31	4	0	0

$$y_0^e = 0.98, \text{gain}=0.01, T_m = 1$$

$T_p \setminus \bar{g}'$	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8
1	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	31	22
7	0	0	0	0	0	0	0	22	20	3	0
8	0	0	0	0	0	0	25	8	0	0	0

$$y_0^e = 0.98, \text{ gain}=0.01, T_m = 2$$

$T_p \setminus \bar{g}'$	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8
1	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	1	37
6	0	0	0	0	0	0	0	0	43	23	3
7	0	0	0	0	0	0	6	24	3	0	0
8	0	0	0	0	0	0	19	3	0	0	0

$$y_0^e = 0.98, \text{ gain}=0.01, T_m = 3$$

$T_p \setminus \bar{g}'$	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8
1	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	2	40	35
6	0	0	0	0	0	0	0	20	42	2	0
7	0	0	0	0	0	0	31	7	0	0	0
8	0	0	0	0	0	23	2	0	0	0	0

$$y_0^e = 0.98, \text{ gain}=0.01, T_m = 4$$

$T_p \setminus \bar{g}_1$	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8
1	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	1	30
5	0	0	0	0	0	0	0	2	40	38	12
6	0	0	0	0	0	0	29	28	6	0	0
7	0	0	0	0	0	41	18	1	0	0	0
8	0	0	0	0	24	15	1	0	0	0	0

$$y_0^e = 0.98, \text{ gain}=0.01, T_m = 5$$

$T_p \setminus \bar{g}'$	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8
1	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	7	41	45
5	0	0	0	0	0	0	3	40	35	6	1
6	0	0	0	0	2	1	37	12	4	0	0
7	0	0	0	0	12	21	2	0	0	0	0
8	0	0	0	0	19	2	0	0	0	0	0

$$y_0^e = 0.98, \text{ gain} = 0.01, T_m = 6$$

$T_p \setminus \bar{g}'$	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8
1	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	17
4	0	0	0	0	0	0	0	21	40	29	14
5	0	0	0	0	0	0	45	25	6	0	0
6	0	0	0	0	0	41	15	0	0	0	0
7	0	0	0	0	37	11	1	0	0	0	0
8	0	0	0	22	11	1	1	0	0	0	0

$$y_0^e = 0.98, \text{ gain} = 0.01, T_m = 7$$

$T_p \setminus \bar{g}'$	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8
1	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	28	39
4	0	0	0	0	0	0	10	42	21	6	1
5	0	0	0	0	0	40	24	6	0	0	0
6	0	0	0	0	42	16	2	0	0	0	0
7	0	0	0	19	15	2	0	0	0	0	0
8	0	0	0	29	0	0	0	0	0	0	0

$$y_0^e = 0.98, \text{ gain} = 0.01, T_m = 8$$

G.2.1 Additional results with higher values for \bar{g}'

The next two tables show the additional rows and columns (with at least one non-zero value) to the corresponding table above with results continued to be based on 100 replications. Note: column for $\bar{g}' = 0.8$ is included to see continuity to the earlier table.

$T_p \setminus \bar{g}'$	0.8	0.85	0.9	0.95	1.0	1.05	1.1	1.15	1.2	1.25
3	0	0	0	1	21	34	44	44	49	24
4	30	41	52	18	5	3	0	0	0	0
5	12	2	0	0	0	0	0	0	0	0

$$y_0^e = 0.98, \text{ gain} = 0.01, T_m = 5$$

$T_p \setminus \bar{g}'$	0.8	0.85	0.9	0.95	1.0	1.05	1.1	1.15	1.2
3	0	1	24	39	44	47	25	17	10
4	45	28	4	3	0	0	0	0	0
5	1	0	0	0	0	0	0	0	0

$$y_0^e = 0.98, \text{ gain} = 0.01, T_m = 6$$

$T_p \setminus \bar{g}'$	0.8	0.85	0.9	0.95	1.0	1.05	1.1	1.15	1.2
3	17	35	42	36	20	8	2	0	1
4	14	3	0	0	1	0	0	0	0

$$y_0^e = 0.98, \text{ gain} = 0.01, T_m = 7$$

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